

Cp1Ch2 XMQs and MS

(Total: 67 marks)

1. CP2_2019 Q6 . 9 marks - CP1ch2 Argand diagrams
2. CP2_2020 Q2 . 9 marks - CP1ch2 Argand diagrams
3. CP2_2021 Q1 . 5 marks - CP1ch2 Argand diagrams
4. CP(AS)_2018 Q3 . 9 marks - CP1ch2 Argand diagrams
5. CP(AS)_2020 Q2 . 8 marks - CP1ch2 Argand diagrams
6. CP(AS)_2020 Q10. 7 marks - CP1ch2 Argand diagrams
7. CP(AS)_2021 Q5 . 10 marks - CP1ch2 Argand diagrams
8. CP(AS)_2022 Q2 . 10 marks - CP1ch2 Argand diagrams

Question	Scheme	Marks	AOs
6(a)	<p>Examples:</p> $\begin{pmatrix} \cos 120 & -\sin 120 \\ \sin 120 & \cos 120 \end{pmatrix} \begin{pmatrix} 6 \\ 2 \end{pmatrix} = \dots \text{or } (6+2i) \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i \right)$ <p>or $\sqrt{40} \left(\cos \arctan \left(\frac{2}{6} \right) + i \sin \arctan \left(\frac{2}{6} \right) \right) \left(\cos \left(\frac{2\pi}{3} \right) + i \sin \left(\frac{2\pi}{3} \right) \right)$</p> <p>or</p> $\sqrt{40} \left(\cos \left(\arctan \left(\frac{2}{6} \right) + \frac{2\pi}{3} \right) + i \sin \left(\arctan \left(\frac{2}{6} \right) + \frac{2\pi}{3} \right) \right)$ <p>or</p> $\sqrt{40} e^{i \arctan \left(\frac{2}{6} \right)} e^{i \left(\frac{2\pi}{3} \right)}$	M1	3.1a
	$(-3 - \sqrt{3}) \text{ or } (3\sqrt{3} - 1)i$	A1	1.1b
	$(-3 - \sqrt{3}) + (3\sqrt{3} - 1)i$	A1	1.1b
	<p>Examples:</p> $\begin{pmatrix} \cos 240 & -\sin 240 \\ \sin 240 & \cos 240 \end{pmatrix} \begin{pmatrix} 6 \\ 2 \end{pmatrix} = \dots \text{or } (6+2i) \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i \right)$ <p>or</p> $\sqrt{40} \left(\cos \arctan \left(\frac{2}{6} \right) + i \sin \arctan \left(\frac{2}{6} \right) \right) \left(\cos \left(\frac{4\pi}{3} \right) + i \sin \left(\frac{4\pi}{3} \right) \right)$ <p>or</p> $\sqrt{40} \left(\cos \left(\arctan \left(\frac{2}{6} \right) + \frac{4\pi}{3} \right) + i \sin \left(\arctan \left(\frac{2}{6} \right) + \frac{4\pi}{3} \right) \right)$ <p>or</p> $\sqrt{40} e^{i \arctan \left(\frac{2}{6} \right)} e^{i \left(\frac{4\pi}{3} \right)}$	M1	3.1a
	$(-3 + \sqrt{3}) \text{ or } (-3\sqrt{3} - 1)i$	A1	1.1b
	$(-3 + \sqrt{3}) + (-3\sqrt{3} - 1)i$	A1	1.1b
	(6)		
(b) Way 1	$\text{Area } ABC = 3 \times \frac{1}{2} \sqrt{6^2 + 2^2} \sqrt{6^2 + 2^2} \sin 120^\circ$ <p>or</p> $\text{Area } AOB = \frac{1}{2} \sqrt{6^2 + 2^2} \sqrt{6^2 + 2^2} \sin 120^\circ$	M1	2.1
	$\text{Area } DEF = \frac{1}{4} ABC \text{ or } \frac{3}{4} AOB$	dM1	3.1a
	$= \frac{3}{8} \times 40 \times \frac{\sqrt{3}}{2} = \frac{15\sqrt{3}}{2}$	A1	1.1b
		(3)	

(b) Way 2	$D\left(\frac{3-\sqrt{3}}{2}, \frac{3\sqrt{3}+1}{2}\right)$ $OD = \sqrt{\left(\frac{3-\sqrt{3}}{2}\right)^2 + \left(\frac{3\sqrt{3}+1}{2}\right)^2} = \sqrt{10}$ $\text{Area } DOF = \frac{1}{2}\sqrt{10}\sqrt{10}\sin 120^\circ$	M1	2.1
	$\text{Area } DEF = 3DOF$	dM1	3.1a
	$= 3 \times \frac{1}{2} \times \sqrt{10}\sqrt{10} \times \frac{\sqrt{3}}{2} = \frac{15\sqrt{3}}{2}$	A1	1.1b
(b) Way 3	$AB = \sqrt{(9+\sqrt{3})^2 + (3-3\sqrt{3})^2} = \sqrt{120}$ $\text{Area } ABC = \frac{1}{2}\sqrt{120}\sqrt{120}\sin 60^\circ (= 30\sqrt{3})$	M1	2.1
	$\text{Area } DEF = \frac{1}{4}ABC$	dM1	3.1a
	$= \frac{1}{4} \times 30\sqrt{3} = \frac{15\sqrt{3}}{2}$	A1	1.1b
(b) Way 4	$D\left(\frac{3-\sqrt{3}}{2}, \frac{3\sqrt{3}+1}{2}\right), E(-3, -1), F\left(\frac{3+\sqrt{3}}{2}, \frac{-3\sqrt{3}+1}{2}\right)$ $DE = \sqrt{\left(\frac{3-\sqrt{3}}{2}+3\right)^2 + \left(\frac{3\sqrt{3}+1}{2}+1\right)^2} (= \sqrt{30})$ $\text{Area } DEF = \frac{1}{2}\sqrt{30}\sqrt{30}\sin 60^\circ$	M1	2.1
	$= \frac{15\sqrt{3}}{2}$	dM1	3.1a
	$= \frac{15\sqrt{3}}{2}$	A1	1.1b
(b) Way 5	$\text{Area } ABC = \frac{1}{2} \begin{vmatrix} 6 & -3-\sqrt{3} & \sqrt{3}-3 & 6 \\ 2 & 3\sqrt{3}-1 & -3\sqrt{3}-1 & 2 \end{vmatrix} = 30\sqrt{3}$	M1	2.1
	$\text{Area } DEF = \frac{1}{4}ABC$	dM1	3.1a
	$= \frac{1}{4} \times 30\sqrt{3} = \frac{15\sqrt{3}}{2}$	A1	1.1b

(9 marks)

Notes

(a)

M1: Identifies a suitable method to rotate the given point by 120° (or equivalent) about the origin. May see equivalent work with modulus/argument or exponential form e.g. an attempt to multiply

by $\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}$ or $e^{\frac{2\pi i}{3}}$

A1: Correct real part or correct imaginary part

A1: Completely correct complex number

M1: Identifies a suitable method to rotate the given point by 240° (or equivalent e.g. rotate their B by 120°) about the origin

May see equivalent work with modulus/argument or exponential form e.g. an attempt to multiply $6 + 2i$ by $\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}$ or $e^{\frac{4\pi}{3}i}$ or their B by $\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}$ or $e^{\frac{2\pi}{3}i}$

A1: Correct real part or correct imaginary part

A1: Completely correct complex number

(b)

In general, the marks in (b) should be awarded as follows:

M1: Attempts to find the area of a relevant triangle

dM1: completes the problem by multiplying by an appropriate factor to find the area of DEF

Dependent on the first method mark

A1: Correct exact area

In some cases it may not be possible to distinguish the 2 method marks. In such cases they can be awarded together for a direct method that finds the area of DEF

Examples:

Way 1

M1: A correct strategy for the area of a relevant triangle such as ABC or AOB

dM1: Completes the problem by linking the area of DEF correctly with ABC or with AOB

A1: Correct value

Way 2

M1: A correct strategy for the area of a relevant triangle such as DOF

dM1: Completes the problem by linking the area of DEF correctly with DOF

A1: Correct value

Way 3

M1: A correct strategy for the area of a relevant triangle such as ABC

dM1: Completes the problem by linking the area of DEF correctly with ABC

A1: Correct value

Way 4

M1dM1: A correct strategy for the area of DEF . Finds 2 midpoints and attempts one side of DEF and uses a correct triangle area formula. By implication this scores both M marks.

A1: Correct value

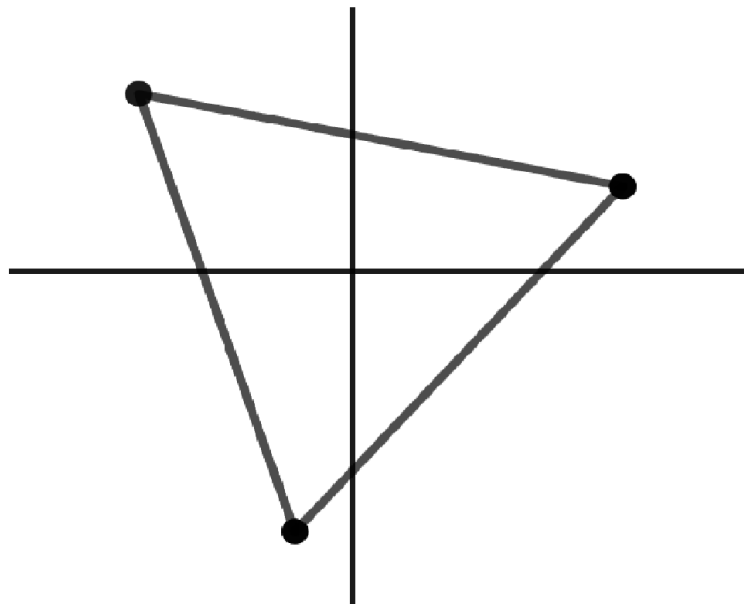
Way 5

M1: A correct strategy for the area of ABC using the “shoelace” method.

dM1: Completes the problem by linking the area of DEF correctly with ABC

A1: Correct value

Note the marks in (b) can be scored using inexact answers from (a) and the A1 scored if an exact area is obtained.



Question	Scheme	Marks	AOs
2(a)	Centre of circle C is $(1, -1)$	B1	1.1b
	$r = \sqrt{(5-1)^2 + (-4+1)^2} = 5$ or $r = \sqrt{(-3-1)^2 + (2+1)^2} = 5$ or $r = \frac{1}{2}\sqrt{(-3-5)^2 + (2+4)^2} = 5$	M1	3.1a
	$ z - 1 + i = 5$ or $ z - (1-i) = 5$	A1	2.5
		(3)	
(b)	$(x-1)^2 + (y+1)^2 = 25, \quad (x-2)^2 + (y-3)^2 = 4$ $x^2 - 2x + 1 + y^2 + 2y + 1 = 25$ $x^2 - 4x + 4 + y^2 - 6y + 9 = 4$ $\Rightarrow 2x + 8y = 32$	M1	3.1a
	$(16-4y)^2 - 4(16-4y) + 4 + y^2 - 6y + 9 = 4$ or $x^2 - 4x + 4 + \left(\frac{16-x}{4}\right)^2 - 6\left(\frac{16-x}{4}\right) + 9 = 4$	M1	1.1b
	$17y^2 - 118y + 201 = 0$ or $17x^2 - 72x + 16 = 0$	A1	1.1b
	$17y^2 - 118y + 201 = 0 \Rightarrow (17y - 67)(y - 3) = 0 \Rightarrow y = \frac{67}{17}, 3$ or $17x^2 - 72x + 16 = 0 \Rightarrow (17x - 4)(x - 4) = 0 \Rightarrow x = \frac{4}{17}, 4$	M1	1.1b
	$y = \frac{67}{17}, 3 \Rightarrow x = \frac{4}{17}, 4$ or $x = \frac{4}{17}, 4 \Rightarrow y = \frac{67}{17}, 3$	M1	2.1
	$4 + 3i, \frac{4}{17} + \frac{67}{17}i$	A1	2.2a
		(6)	

(9 marks)

Notes

(a)

B1: Correct coordinates of centre

M1: Fully correct strategy for identifying the radius. If the diameter is calculated this must be halved to achieve this mark.

A1: Correct equation using the required notation

(b)

M1: Begins the process of finding z_1 and z_2 by using the Cartesian equations to obtain the equation of the line of intersection

M1: Substitutes back into the equation of one of the circles to obtain an equation in one variable

A1: Correct 3 term quadratic

M1: Solves their 3TQ

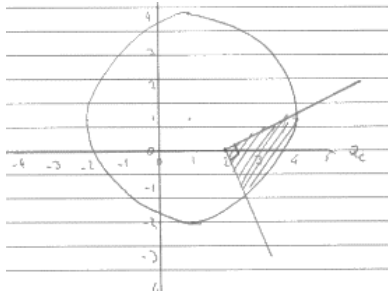
M1: Substitutes to find values of the other variable to complete the process of finding z_1 and z_2

A1: Correct complex numbers

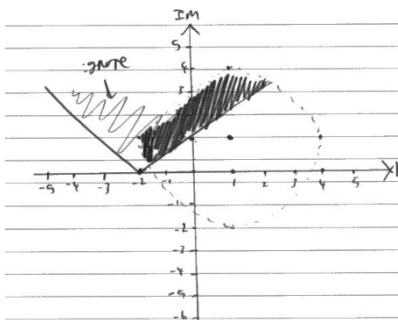
Question	Scheme	Marks	AOs
1(a) (i)	$ z_1 z_2 = 3\sqrt{2}$	B1	1.1b
	$\arg(z_1 z_2) = \frac{\pi}{3} + \left(-\frac{\pi}{12}\right) = \frac{\pi}{4}$ o.e.	B1	1.1b
		(2)	
(b) (i)	$n = 8$	B1ft	2.2a
	$ w^n = (\text{'their } z_1 z_2 \text{'})^{\text{their } n}$	M1	1.1b
	$ w^n = 104\,976$	A1	1.1b
		(3)	
(5 marks)			
Notes:			
<p>(a)</p> <p>(i)</p> <p>B1: Deduces $z_1 z_2 = 3\sqrt{2}$</p> <p>(ii)</p> <p>B1: Deduces $\arg(z_1 z_2) = \frac{\pi}{4}$ o.e</p> <p>These marks may be awarded for $z_1 z_2 = 3\sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$</p>			
<p>(b)</p> <p>(i)</p> <p>B1ft: 2π divided by their $\arg(z_1 z_2)$ found in part (a) (ii) to give an integer</p> <p>Alternatively smallest positive integer multiple required to make their argument a multiple of 2π</p> <p>(ii)</p> <p>M1: Their answer to (a) (i) to the power of their n.</p> <p>A1: 104 976</p>			

Question	Scheme	Marks	AOs
3(a)		M1	1.1b
		M1	1.1b
		A1	2.2a
		M1	3.1a
		A1	1.1b
		(5)	
(b)	$(x-1)^2 + (y-1)^2 = 9, y = x - 2 \Rightarrow x = \dots, \text{ or } y = \dots$	M1	3.1a
	$x = 2 + \frac{\sqrt{14}}{2}, y = \frac{\sqrt{14}}{2}$	A1	1.1b
	$ w ^2 = \left(2 + \frac{\sqrt{14}}{2}\right)^2 + \left(\frac{\sqrt{14}}{2}\right)^2$	M1	1.1b
	$= 11 + 2\sqrt{14}$	A1	1.1b
		(4)	
(9 marks)			
Notes			
<p>(a)</p> <p>M1: Circle or arc of a circle with centre in first quadrant and with the circle in all 4 quadrants or arc of circle in quadrants 1 and 2</p> <p>M1: A “V” shape i.e. with both branches above the x-axis and with the vertex on the positive real axis. Ignore any branches below the x-axis.</p> <p>A1: Two half lines that meet on the positive real axis where the right branch intersects the circle or arc of a circle in the first quadrant and the left branch intersects the circle or arc of a circle in the second quadrant but not on the y-axis.</p> <p>M1: Shades the region between the half-lines and within the circle</p> <p>A1: Cso. A fully correct diagram including 2 marked (or implied by ticks) at the vertex on the real axis with the correct region shaded and all the previous marks scored.</p> <p>(b)</p> <p>M1: Identifies a suitable strategy for finding the x or y coordinate of the point of intersection.</p> <p>Look for an attempt to solve equations of the form $(x \pm 1)^2 + (y \pm 1)^2 = 9$ or 3 and $y = \pm x \pm 2$</p> <p>A1: Correct coordinates for the intersection (there may be other points but allow this mark if the correct coordinates are seen). (The correct coordinates may be implied by subsequent work.)</p> <p>Allow equivalent exact forms and allow as a complex number e.g. $2 + \frac{\sqrt{14}}{2} + \frac{\sqrt{14}}{2}i$</p> <p>M1: Correct use of Pythagoras on their coordinates (There must be no i's)</p> <p>A1: Correct exact value by cso</p> <p>Note that solving $(x-1)^2 + (y-1)^2 = 9, y = x + 2$ gives $x = \frac{\sqrt{14}}{2}, y = 2 + \frac{\sqrt{14}}{2}$ and hence the correct answer fortuitously so scores M1A0M1A0</p>			

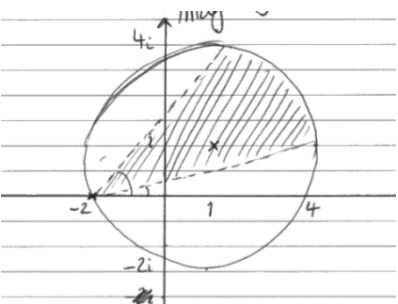
Example marking for 3(a)



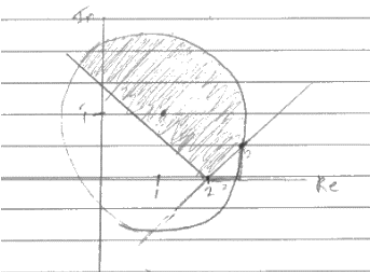
M1: Circle with centre in first quadrant
 M0: The branches of the "V" must be above the x-axis
 A0: Follows M0
 M1: Shades the region between the half-lines and within the circle
 A0: Depends on all previous marks



M1: Circle with centre in first quadrant
 M0: The vertex of the "V" must be on the positive x-axis
 A0: Follows M0
 M1: Shades the region between the half-lines and within the circle (BOD)
 A0: Depends on all previous marks



M1: Circle with centre in first quadrant
 M0: The vertex of the "V" must be on the positive x-axis
 A0: Follows M0
 M1: Shades the region between the half-lines and within the circle
 A0: Depends on all previous marks



M1: Circle with centre in first quadrant
 M1: A "V" shape i.e. with both branches above the x-axis and with the vertex on the positive real axis. Ignore any branches below the x-axis.
 A1: Two half lines that meet on the positive real axis where the right branch intersects the circle in the first quadrant and the left branch intersects the circle in the second quadrant.
 M1: Shades the region between the half-lines and within the circle
 A1: A fully correct diagram including 2 marked at the vertex on the real axis with the correct region shaded and all the previous marks scored.

Question	Scheme	Marks	AOs
2(a)	$ z_1 = \sqrt{13}$ and $\arg z_1 = \tan^{-1}\left(\frac{3}{2}\right)$	B1	1.1b
	$z_1 = \sqrt{13}(\cos 0.9828 + i \sin 0.9828)$	B1ft	1.1b
		(2)	
(b)	A complete method to find the modulus of z_2 e.g. $ z_1 = \sqrt{13}$ and uses $ z_1 z_2 = z_1 \times z_2 = 39\sqrt{2} \Rightarrow z_2 = 3\sqrt{26}$ or $\sqrt{234}$	M1 A1	3.1a 1.1b
	A complete method to find the argument of z_2 e.g. $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2) = \frac{\pi}{4} \Rightarrow \arg(z_2) = \dots$ $\arg(z_2) = \frac{\pi}{4} - \tan^{-1}\left(\frac{3}{2}\right)$ or $\frac{\pi}{4} - 0.9828$ or $-0.1974\dots$	M1 A1	3.1a 1.1b
	$z_2 = 3\sqrt{26}(\cos(' - 0.1974\dots ') + i \sin(' - 0.1974\dots '))$ or $z_2 = a + bi \Rightarrow a^2 + b^2 = 234$ and $\tan^{-1}(-0.1974) = \frac{b}{a} \Rightarrow \frac{b}{a} = -0.2$ $\Rightarrow a = \dots$ and $b = \dots$	ddM1	1.1b
	Deduces that $z_2 = 15 - 3i$ only	A1	2.2a
	Alternative $z_1 z_2 = (a + bi)(2 + 3i) = (2a - 3b) + (3a + 2b)i$		
	$(2a - 3b)^2 + (3a + 2b)^2 = (39\sqrt{2})^2$ or 3042 $\Rightarrow a^2 + b^2 = 234$ or $ z_1 z_2 = z_1 \times z_2 = 39\sqrt{2} \Rightarrow z_2 = 3\sqrt{26}$ or $\sqrt{234}$ $\Rightarrow a^2 + b^2 = 234$	M1 A1	3.1a 1.1b
	$\arg[(2a - 3b) + (3a + 2b)i] = \frac{\pi}{4} \Rightarrow \tan^{-1}\left(\frac{3a + 2b}{2a - 3b}\right) = \frac{\pi}{4} \Rightarrow \frac{3a + 2b}{2a - 3b} = 1$ $\Rightarrow a = -5b$	M1 A1	3.1a 1.1b
	Solves $a = -5b$ and $a^2 + b^2 = 234$ to find values for a and b	ddM1	1.1b
	Deduces that $z_2 = 15 - 3i$ only	A1	2.2a
		(6)	
(8 marks)			

Notes:**(a)**

B1: Correct exact value for $|z_1| = \sqrt{13}$ and $\arg z_1 = \tan^{-1}\left(\frac{3}{2}\right)$. The value for $\arg z_1$ can be implied by sight of awrt 0.98 or awrt 56.3°

B1ft: Follow through on $r = |z_1|$ and $\theta = \arg z_1$ and writes $z_1 = r(\cos \theta + i \sin \theta)$ where r is exact and θ is correct to 4 s.f. do not follow through on rounding errors.

(b)

M1: A complete method to find the modulus of z_2

A1: $|z_2| = 3\sqrt{26}$

M1: A complete method to find the argument of z_2

A1: $\arg(z_2) = \frac{\pi}{4} - \tan^{-1}\left(\frac{3}{2}\right)$ or $\frac{\pi}{4} - 0.9828$ or $-0.1974\dots$

ddM1: Writes z_2 in the form $r(\cos \theta + i \sin \theta)$, dependent on both previous M marks.

Alternative forms two equations involving a and b using the modulus and argument of z_2 and solve to find values for a and b

A1: Deduces that $z_2 = 15 - 3i$ only

(b) Alternative: $z_1 z_2 = (a + bi)(2 + 3i) = (2a - 3b) + (3a + 2b)i$

M1: A complete method to find an equation involving a and b using the modulus

A1: Correct simplified equation $a^2 + b^2 = 234$ o.e.

M1: A complete method to find an equation involving a and b using the argument.

Note $\tan^{-1}\left(\frac{2a - 3b}{3a + 2b}\right) = \frac{\pi}{4}$ this would score **M0 A0 ddM0 A0**

A1: Correct simplified equation $a = -5b$ o.e.

ddM1: Dependent on both the previous method marks. Solves their equations to find values for a and b

A1: Deduces that $z_2 = 15 - 3i$ only

10. Given that there are two distinct complex numbers z that satisfy

$$\{z: |z - 3 - 5i| = 2r\} \cap \left\{z: \arg(z - 2) = \frac{3\pi}{4}\right\}$$

determine the exact range of values for the real constant r .

(7)

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Question	Scheme	Marks	AOs
10	$(x-3)^2 + (y-5)^2 = (2r)^2$ and $y = -x + 2$	B1	1.1b
	$(x-3)^2 + (-x+2-5)^2 = (2r)^2$ or $(-y+2-3)^2 + (y-5)^2 = (2r)^2$	M1	3.1a
	$2x^2 + 18 - 4r^2 = 0$ or $2y^2 - 8y + 26 - 4r^2 = 0$	A1	1.1b
	$b^2 - 4ac > 0 \Rightarrow 0^2 - 4(2)(18 - 4r^2) > 0 \Rightarrow r > \dots$ or $x^2 = 9 - 2r^2 \Rightarrow 9 - 2r^2 > 0 \Rightarrow r > \dots$ or $b^2 - 4ac > 0 \Rightarrow (-8)^2 - 4(2)(26 - 4r^2) > 0 \Rightarrow r > \dots$	dM1	3.1a
	Finds a maximum value for r $(2r)^2 = 5^2 + (3-2)^2 \Rightarrow r = \dots$	M1	3.1a
	$\frac{3\sqrt{2}}{2} < r < \frac{\sqrt{26}}{2}$ o.e.	A1 A1	1.1b 1.1b
	Alternative Using a circle with centre (3, 5) and radius $2r$ and $y = -x + 2$	B1	1.1b
	$y - 5 = 1(x - 3) \Rightarrow y = x + 2$ $x + 2 = -x + 2 \Rightarrow x = \dots$	M1	3.1a
	(0, 2)	A1	1.1b
	$2r > \sqrt{(3-0)^2 + (5-2)^2} \Rightarrow r > \dots$	dM1	3.1a
	Finds a maximum value for r $(2r)^2 = 5^2 + (3-2)^2 \Rightarrow r = \dots$	M1	3.1a
	$\frac{3\sqrt{2}}{2} < r < \frac{\sqrt{26}}{2}$ o.e.	A1 A1	1.1b 1.1b
		(7)	

(7 marks)

Notes:**B1:** Correct equations for each loci of points**M1:** A complete method to find a 3TQ involving one variable using equations of the form $(x \pm 3)^2 + (y \pm 5)^2 = (2r)^2$ or $2r^2$ or r^2 and $y = \pm x \pm 2$ **A1:** Correct quadratic equation

dM1: Dependent on previous method mark. A complete method uses $b^2 - 4ac > 0$ or rearranges to find $x^2 = f(r)$ and uses $f(r) > 0$ to the minimum value of r .

M1: Realises there will be an upper limit for r and uses Pythagoras theorem

$$(2r)^2 = (y \text{ coord of centre})^2 + (x \text{ coord of centre} - 2)^2$$

$$\text{condone } (r)^2 = (y \text{ coord of centre})^2 + (x \text{ coord of centre} - 2)^2$$

A1: One correct limit, either $\frac{3\sqrt{2}}{2} < r$ or $r < \frac{\sqrt{26}}{2}$ o.e.

A1: Fully correct inequality

Alternative

B1: Using a circle with centre (3, 5) and radius $2r$ and $y = -x + 2$

M1: A complete method to find the point of intersection of the line $y = \pm x \pm 2$ and circle where the line is a tangent to the circle.

A1: Correct point of intersection

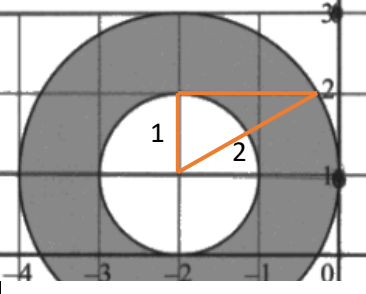
dM1: Finds the distance between the point of intersection and the centre and uses this to find the minimum value of r . Condone radius of r .

M1: Realises there will be an upper limit for r and uses Pythagoras theorem

$$(2r)^2 = (y \text{ coord of centre})^2 + (x \text{ coord of centre} - 2)^2$$

A1: One correct limit, either $\frac{3\sqrt{2}}{2} < r$ or $r < \frac{\sqrt{26}}{2}$ o.e.

A1: Fully correct inequality

Question	Scheme	Marks	AOs
5(a)	$a = 1, d = 2$	B1	1.1b
	$b = 2$	B1	1.1b
	$c = -1$	B1	1.1b
		(3)	
(b)	$ z - i = z - 3i \Rightarrow y = 2$	B1	2.2a
	Area between the circles = $\pi \times 2^2 - \pi \times 1^2$	M1	1.1a
	 <p>Angle subtended at centre = $2 \times \cos^{-1}\left(\frac{1}{2}\right)$ Alternatively $(x+2)^2 + (y-1)^2 = 4, y = 2 \Rightarrow x = \dots$ Or $x = \sqrt{2^2 - 1^2}$ Leading to Angle subtended at centre = $2 \times \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$</p>	M1	3.1a
	Segment area = $\frac{1}{3} \times \pi \times 2^2 - \frac{1}{2} \times 2^2 \times \sin\left(\frac{2\pi}{3}\right) \left\{ = \frac{4}{3}\pi - \sqrt{3} \right\}$	M1 A1	2.1 1.1b
	Area of Q: $\pi \times 2^2 - \pi \times 1^2 - \left(\frac{1}{3} \times \pi \times 2^2 - \frac{1}{2} \times 2^2 \times \sin\left(\frac{2\pi}{3}\right) \right)$	M1	3.1a
	$= \frac{5\pi}{3} + \sqrt{3}$	A1	1.1b
		(7)	

(10 marks)

Notes

(a)

B1: Correct values for a and d B1: Correct value for b B1: Correct value for c

(b)

B1: Deduces that $|z - i| = |z - 3i|$ is a perpendicular bisector with equation $y = 2$, this may be drawn on a diagram.

M1: Selects the correct procedure to find the area of the large circle – the area of the small circle.

M1: Correct method to find the angle at the centre (or half this angle).

Recognises that the hypotenuse is the radius of the larger circle and the adjacent is the radius of the smaller circle and using cosine

Alternatively find where the perpendicular bisector intersects the larger circle so uses their $y = 2$ and the equation of the larger circle in an attempt to establish the x values for the intersection points or uses geometry and Pythagoras to identify the required length and then uses tangent.

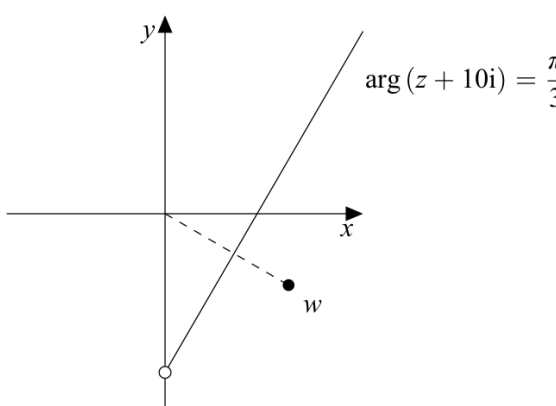
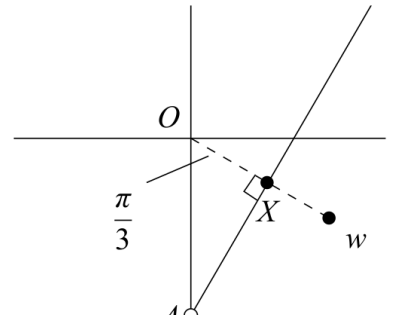
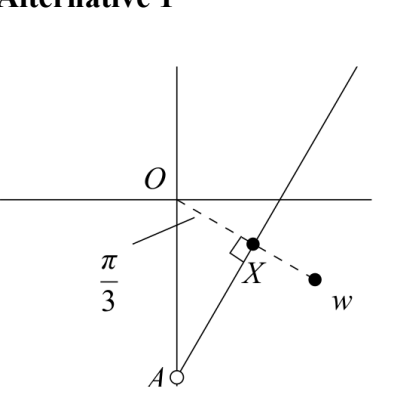
M1: Correct method for the area of the minor segment (allow equivalent work)

A1: Correct expression

M1: Fully correct strategy for the required area. Must be subtracting the area of the minor segment from the annulus area.

A1: Correct exact answer

Note: 6.968

Question	Scheme	Marks	AOs	
2(a)	$ w = \sqrt{(4\sqrt{3})^2 + (-4)^2} = 8$	B1	1.1b	
	$\arg w = \arctan\left(\frac{\pm 4}{4\sqrt{3}}\right) = \arctan\left(\pm \frac{1}{\sqrt{3}}\right)$	M1	1.1b	
	$= -\frac{\pi}{6}$	A1	1.1b	
	So $(w =) 8\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$	A1	1.1b	
	(4)			
(b)		(i) w in 4 th quadrant with either $(4\sqrt{3}, -4)$ seen or $-\frac{\pi}{4} < \arg w < 0$	B1	1.1b
		(ii) half line with positive gradient emanating from imaginary axis.	M1	1.1b
		The half line should pass between O and w starting from a point on the imaginary axis below w	A1	1.1b
(3)				
(c)		ΔOAX is right angled at X so $OX = 10 \sin \frac{\pi}{6} = 5$ (oe)	M1	3.1a
		So shortest distance is $WX = OW - OX = '8' - 5 = \dots$	M1	1.1b
		So min distance is 3	A1	1.1b
Alternative 1		A complete method to find the coordinates of X . Finds the equation of the line from O to w , $y = -\frac{1}{\sqrt{3}}x$ and the equation of the half line $y = \sqrt{3}x - 10$, solves to find the point of intersection $X\left(\frac{5\sqrt{3}}{2}, -\frac{5}{2}\right)$	M1	3.1a
		Finds the length WX $\sqrt{\left(4\sqrt{3} - \frac{5\sqrt{3}}{2}\right)^2 + \left(-4 - -\frac{5}{2}\right)^2}$	M1	1.1b
		So min distance is 3	A1	1.1b
Alternative 2		M1	3.1a	

	<p>Finds the length $AW = \sqrt{(4\sqrt{3}-0)^2 + (-4--10)^2} = \dots\{\sqrt{84}\}$</p> <p>Finds the angle between the horizontal and the line AW</p> $= \tan^{-1}\left(\frac{-4--10}{4\sqrt{3}}\right) = \dots\{0.7137\dots\text{radians or } 40.89\dots^\circ\}$		
	<p>Finds the length of $WX = \sqrt{84} \times \sin\left(\frac{\pi}{3} - 0.7137\right) = \dots$</p> <p>Or $= \sqrt{84} \times \sin(60 - 40.89) = \dots$</p>	M1	1.1b
	So min distance is 3	A1	1.1b
	<p>Alternative 3</p> <p>Vector equation of the half line $r = \begin{pmatrix} 0 \\ -10 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$</p> $XW = \begin{pmatrix} 4\sqrt{3} - \lambda \\ -4 - \lambda\sqrt{3} - (-10) \end{pmatrix}$ <p>Then either</p> $\begin{pmatrix} 4\sqrt{3} - \lambda \\ 6 - \lambda\sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} = 4\sqrt{3} - \lambda + 6\sqrt{3} - 3\lambda = 0 \Rightarrow \lambda = \dots\left\{\frac{5}{2}\sqrt{3}\right\}$ $r = \begin{pmatrix} 0 \\ -10 \end{pmatrix} + \frac{5}{2}\sqrt{3} \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} = \dots$ <p>Or $XW^2 = (4\sqrt{3} - \lambda)^2 + (6 - \lambda\sqrt{3})^2 = 48 - 8\lambda\sqrt{3} + \lambda^2 + 36 - 12\lambda\sqrt{3} + 3\lambda^2$</p> <p>$xw^2 = 84 - 20\lambda\sqrt{3} + 4\lambda^2$ leading to</p> $\frac{d(XW^2)}{d\lambda} = -20\sqrt{3} + 8\lambda = 0 \Rightarrow \lambda = \dots$	M1	3.1a
	<p>Finds the length $WX = \sqrt{\left(4\sqrt{3} - \frac{5\sqrt{3}}{2}\right)^2 + \left(-4 - \frac{5}{2}\right)^2}$</p> <p>Or $XW = \sqrt{\left(4\sqrt{3} - \frac{5}{2}\sqrt{3}\right)^2 + \left(6 - \frac{5}{2}\sqrt{3}\right)^2}$</p>	M1	1.1b
	So min distance is 3	A1	1.1b
		(3)	
(10 marks)			
Notes:			
<p>(a)</p> <p>B1: Correct modulus</p> <p>M1: Attempts the argument. Allow for $\arctan\left(\frac{\pm 4}{\pm 4\sqrt{3}}\right)$ or equivalents using the modulus (may be in wrong quadrant for this mark).</p> <p>A1: Correct argument $-\frac{\pi}{6}$ (must be in fourth quadrant but accept $\frac{11\pi}{6}$ or other difference of 2π for this mark).</p>			

A1: Correct expression found for w , in the correct form, must have positive $r = 8$ and $\theta = -\frac{\pi}{6}$.

Note: using degrees B1 M1 A0 A0

(b)(i)&(ii)

B1: w plotted in correct quadrant with either the correct coordinate clearly seen or above the line $y = -x$

M1: Half line drawn starting on the imaginary axis away from O with positive gradient (need not be labelled)

A1: Sketch on **one diagram**— both previous marks must have been scored and the half line should pass between O and w starting from a point on the imaginary axis below w . (You may assume it starts at $-10i$ unless otherwise stated by the candidate)

Note: If candidates draw the loci on separate diagrams the maximum they can score is B1 M1 A0

(c)

M1: Formulates a correct strategy to find the shortest distance, e.g. uses right angle OXA where X is where the lines meet and proceeds at least as far as OX .

M1: Full method to achieve the shortest distance, e.g. for $WX = OW - OX$.

A1: **cao** shortest distance is 3

Alternative 1:

M1: Uses a correct method to find the equation of the line from O to w , $y = -\frac{1}{\sqrt{3}}x$ and the equation of the half line $y = \sqrt{3}x - 10$, solves to find the point of intersection $X\left(\frac{5\sqrt{3}}{2}, -\frac{5}{2}\right)$

If the incorrect gradient(s) is used with no valid method seen this is M0

M1: Finds the length $WX = \sqrt{\left(\text{their } \frac{5\sqrt{3}}{2} - 4\sqrt{3}\right)^2 + \left(\text{their } -\frac{5}{2} - -4\right)^2} = \dots$ condone a sign slip in the brackets.

A1: **cao** shortest distance is 3

Alternative 2:

M1: Uses a correct method to find the length AW and a correct method to find the angle between the horizontal and the line AW

M1: Finds the length of $WX = \text{their } \sqrt{84} \times \sin\left(\frac{\pi}{3} - \text{their } 0.7137\right) = \dots$

A1: **cao** shortest distance is 3

Alternative 3

M1: Finds the vector equation of the half line, then XW .

Then either: Sets dot product XW and the line $= 0$ and solves for λ . Substitutes their λ into the equation of the half line to find the point of intersection.

Or finds the length of XW and differentiates, set $= 0$ and solve for λ

M1: Finds the length $WX = \sqrt{\left(\text{their } \frac{5\sqrt{3}}{2} - 4\sqrt{3}\right)^2 + \left(\text{their } -\frac{5}{2} - -4\right)^2} = \dots$ condone a sign slip in the brackets.

Or substitutes their value for λ into the length of (d)

A1: **cao** shortest distance is 3