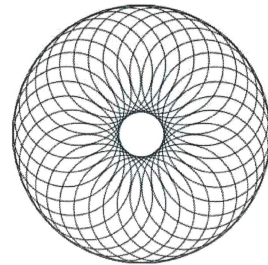


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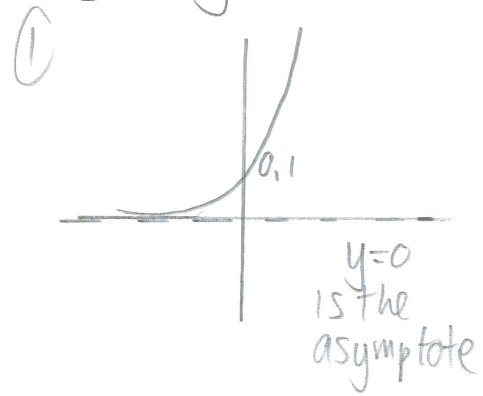
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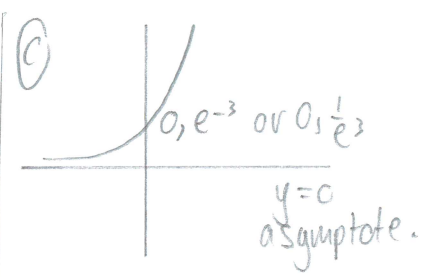
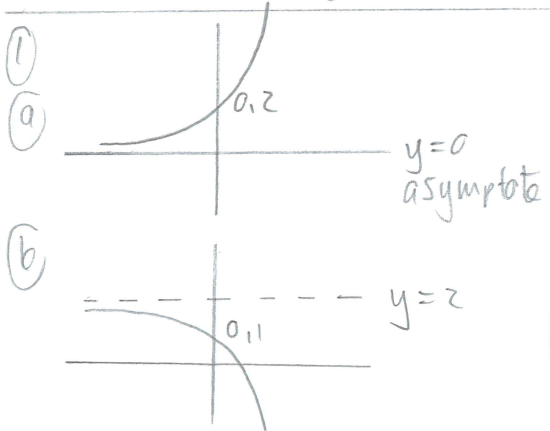
<p>(1) Indices            (2) Expanding Brackets            (3) Factorising Expressions            (4) More Indices (Negative and Fractional)            (5) Working with Surds            (6) Solving Quadratic Equations            (7) Completing the Square for Quadratics Expressions            (8) Function Notation            (9) Sketching Quadratic Graphs            (10) The Discriminant for Quadratic Equations            (11) Applications of Quadratics Equations            (12) Solving Linear Simultaneous Equations            (13) Linear &amp; Non-Linear Simultaneous Equations            (14) Graphing Simultaneous Equations            (15) Linear Inequalities            (16) Quadratic Inequalities            (17) Graphing Inequalities            (18) Shading Inequalities            (19) Cubic Graphs            (20) Quartic Graphs            (21) Reciprocal Graphs            (22) The Intersection of Graphs            (23) Transforming Graphs (Translations)            (24) Transforming Graphs (Stretching/Reflecting)            (25) Straight Line Graphs in the form <math>y = mx + c</math>            (26) More Straight Line Graphs            (27) Straight Line Graphs (Parallel &amp; Perpendicular)            (28) The Geometry of Straight Lines            (29) The Application of Linear Graphs            (30) Circle Geometry Midpoint &amp; Perpendicular</p>	<p>(31) The Equation of a Circle            (32) Circles and Straight Lines (Intersections)            (33) Circles (Tangents and Chords)            (34) Circles and Triangles            (35) Algebraic Fractions            (36) Polynomial Division            (37) The Factor and Remainder Theorem            (38) An Introduction to Mathematical Proof            (39) Methods of Proof            (40) Binomial Expansion (Using Pascal's Triangle)            (41) Binomial Expansion (Factorial Notation)            (42) Binomial Expansion (The <math>\binom{n}{r}</math> Method)            (43) Binomial Expansion (Problem Solving)            (44) Binomial Expansion (Estimations and Approximations)            (45) The Cosine Rule            (46) The Sine Rule            (47) Areas of a Triangles            (48) Triangles (Problem Solving)            (49) Sine, Cosine &amp; Tangent Graphs            (50) Transforming Graphs (Trigonometry)            (51) The 'CAST' Diagram for Trig Ratios            (52) Trigonometry (Exact Values)            (53) Proving Trigonometric Identities            (54) Solving Basic Trigonometric Equations            (55) More Challenging Trigonometric Equations            (56) Using Identities to Solve Trig Equations            (57) Vectors (Introduction)</p>	<p>(58) Vector Notation (Column and i and j form)            (59) Vectors (Magnitude and Direction)            (60) Vectors (Position and Direction Vectors)            (61) Vector Geometry            (62) Application of Vectors            (63) Differentiation (Gradients of Curves)            (64) Differentiation from 1st Principles            (65) Differentiating <math>x^n</math> (Basic Powers of )            (66) Differentiation (Quadratic Expression)            (67) Differentiation (Multiple Terms)            (68) Differentiation (Gradients, Tangents and Normals)            (69) Differentiation (Increasing and Decreasing Functions)            (70) Differentiation (Stationary Points)            (71) Differentiation (Gradient Functions)            (72) The Applications of Differentiation            (73) Integration (Basic Expressions (<math>x^n</math>))            (74) Indefinite Integrals            (75) Integration (Finding <math>c</math> and Finding Functions)            (76) Integration (Definite Integrals)            (77) Integration (Basic Areas Under Curves)            (78) Integration ('Negative and Positive Areas')            (79) Integration (Areas between Curves and Lines)            (80) Basic Exponential Functions            (81) 'The' Exponential Function <math>y = e^x</math>            (82) Applications of Basic Exponential Models            (83) Logarithms (Simplifying &amp; Evaluating)            (84) Logarithms (The Log Laws)            (85) Logarithms (Log and Exponential Equations)</p>
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(81)  $y = e^x$  Pure  $y^1$



- ② a) 20.09  
b) 60.34  
c) 0.14

③ a)  $\frac{dy}{dx} = e^x$  b)  $\frac{dy}{dx} = 3e^x$   
c)  $\frac{dy}{dx} = 4e^{4x}$  d)  $\frac{dy}{dx} = e^x + 1$   
e)  $\frac{dy}{dx} = -e^x$  f)  $\frac{dy}{dx} = -e^{-x}$

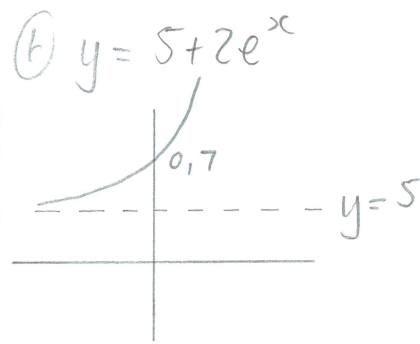


② a)  $f'(x) = 4e^{4x+1}$   
b)  $f'(x) = e^x + 2x$   
c)  $f'(x) = 12e^{3x}$   
d)  $f(x) = e^{2x} - 6e^x$   
 $\therefore f'(x) = 2e^{2x} - 6e^x$

⑥  $f'(x) = 10e^{5x}$   
 $f'(2) = 10e^{10} = 220264.7$

③  $y = (e^x + 1)(e^x + 1)$   
 $= e^{2x} + 2e^x + 1$   
 $\frac{dy}{dx} = 2e^{2x} + 2e^x$  ✓

① a) Substitute points into equation:  
 $5 + \frac{z}{e} = a + \frac{b}{e}$   
 $a = 5, b = z$



①  $y > 5$

②  $x=0$   
 $y = 7 - 5e^{0-2}$   
 $= 7 - 5e^{-2}$   
or  $7 - \frac{5}{e^2}$

$\therefore (0, 7 - \frac{5}{e^2})$

⑥  $f(x) < 7$

①  $f'(x) = 0 - 5e^{x-2} \times 1$   
 $= -5e^{x-2}$

①  $f'(3) = -5e^{3-2}$   
 $= -5e$

③  $\frac{dy}{dx} = 3e^{3x}$   
when  $x=1$   $\frac{dy}{dx} = 3e^3$   
 $\therefore$  normal gradient =  $-\frac{1}{3e^3}$

when  $x=1$   $y = 3e^3$   
Equation of Normal  
 $y - 3e^3 = -\frac{1}{3e^3}(x-1)$

when  $y=0$   
 $-3e^3 = -\frac{1}{3e^3}(x-1)$   
 $9e^6 = x-1$   
 $9e^6 + 1 = x$

when  $x=0$   
 $y - 3e^3 = -\frac{1}{3e^3}(0-1)$   
 $y = \frac{1}{3e^3} + 3e^3$

Points A and B  
 $(9e^6 + 1, 0)$  and  $(0, \frac{1}{3e^3} + 3e^3)$  o.e  
 $(0, \frac{1+9e^6}{3e^3})$  for example

# 82 Exponential Modelling.

(1) When  $t=0$   $N=10$

(a)  $\therefore 10$

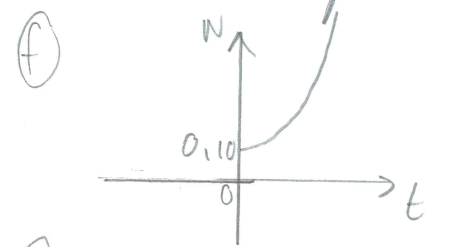
(b)  $N = 10e^{0.2 \times 20}$   
 $= 545.98$

$\therefore \approx 546$

(c)  $\frac{dN}{dt} = 0.2 \times 10e^{0.2t}$   
 $= 2e^{0.2t}$

(d)  $2e^{0.2 \times 8} = 9.906..$

(e) The rate at which the rats are increasing in number per week.



(f) Unlikely that Evan will grow the colony for a large number

of years, hence  $t$  would need restricting

(1) When  $n=0$

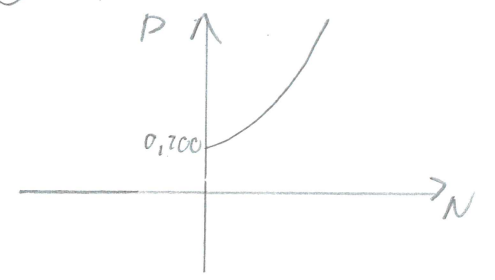
$P = 40e^0 + 160$   
 $= 40(1) + 160$   
 $= 200 \checkmark$

(b)  $P = 40e^{0.1 \times 12} + 160$   
 $= 292.8..$   
 $\therefore \approx 293$

(c)  $\frac{dP}{dn} = 0.1 \times 40e^{0.1n}$   
 $= 4e^{0.1n}$   
 $(K=4)$

(d) The rate at which the population is increasing each year.

(e)  $4e^{0.1 \times 20} = 29.55..$



(2)  $M = 2 + 3e^0$   
 $= 2 + 3$   
 $= 5 \text{ Kg}$

(b) Decay as  $e^{-\frac{t}{8}}$  is a decaying function.

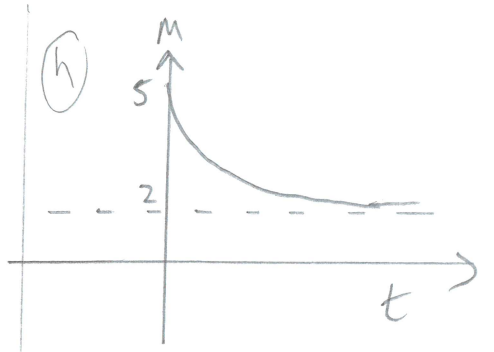
(c)  $M = 2 + 3e^{-\frac{12}{8}}$   
 $M = 1.33.. \text{ Kg}$   
 $\therefore 1300 \text{ g}$

(a)  $\frac{dM}{dt} = 0 + -\frac{1}{8} \times 3e^{-\frac{t}{8}}$   
 $= -\frac{3}{8}e^{-\frac{t}{8}}$   
 $= -0.375e^{-\frac{t}{8}} \checkmark$

(c)  $-0.375e^{-\frac{9}{8}} = -0.1217$

(f) The moss on the rock is decaying at a rate of  $0.122 \text{ Kg/year}$  after 9 years

(g) Yes, min value  $= 2$ . It will never be less than 2



(1) (a)  $\frac{12000}{e^{\frac{1}{4}t}} \rightarrow 0$   
as  $t \rightarrow \infty$

(b) When  $t=0$   $V = 20'000$

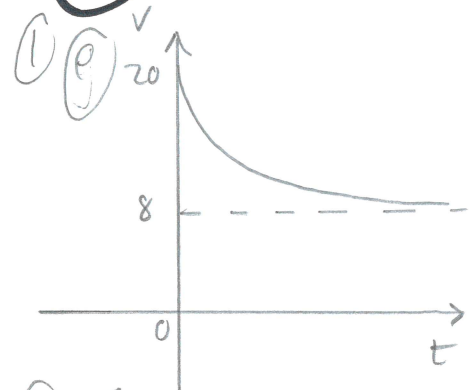
(c)  $V = 8000 + \frac{12000}{e^2}$   
 $V = 9624.023..$   
 $\therefore \underline{\underline{\pounds 9624}}$

(d)  $V = 8000 + 12000e^{-\frac{1}{4}t}$   
 $\frac{dV}{dt} = 0 - 3000e^{-\frac{1}{4}t}$   
when  $t=10$

$\frac{dV}{dt} = -3000e^{-\frac{10}{4}}$   
 $= -246.25$

$\therefore \underline{\underline{\pounds 246 \text{ p/year}}}$

# 82 Continued



(f) The value £8000 is the value the boat will never fall below

(g) The boat is unlikely to be worth £8000 when  $t$  gets very big

(2)  $P = 100 + Ae^{bt}$

(a)  $120 = 100 + Ae^0$

$120 = 100 + A$

$20 = A$

(b)  $P = 100 + 120e^{bt}$

$\frac{dP}{dt} = 0 + 120be^{bt}$

$\therefore 120b = 6$

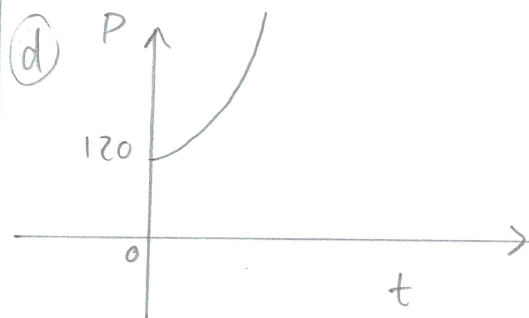
$b = 0.05$

(c)  $P = 100 + 20e^{0.05t}$

(d)  $P = 100 + 20e^{0.05 \times 10}$

$P = 132.97 \dots$

$\therefore 132'974$



(e)  $4 \cdot Q = 6e^{0.05t}$

$\frac{20}{3} = e^{0.05t}$

$\ln\left(\frac{20}{3}\right) = 0.05t$

$t = \frac{\ln\left(\frac{20}{3}\right)}{0.05}$

$t = 37.3y$

(3)  $\frac{dA}{dt} = ce^{dt}$

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Steve Blades

# 83 Logarithms

1) a)  $\log_3 9 = 2$  b)  $\log_5 125 = 3$

c)  $\log_8 64 = 2$  d)  $\log_4 \frac{1}{4} = -1$

e)  $\log_9 1 = 0$  f)  $\log_8 4 = \frac{2}{3}$

2) a)  $2^3 = 8 \therefore 3$

b)  $3^4 = 81 \therefore 4$

c)  $4^2 = 16 \therefore 2$

d)  $5^3 = 125 \therefore 3$

e)  $2^5 = 32 \therefore 5$

f)  $7^1 = 7 \therefore 1$

3) a) 4.75 b) 0.565

c) 1.30

1) a)  $2^3 = x \therefore x = 8$

b)  $3^x = 1 \therefore x = 0$

c)  $4^x = 2 \therefore x = \frac{1}{2}$

d)  $3^x = 3 \therefore x = 1$

e)  $6^x = \frac{1}{36} \therefore x = -2$

f)  $5^x = \frac{1}{5} \therefore x = -1$

g)  $2^4 = x - 1 \therefore x = 17$

h)  $5^4 = 2x \therefore 625 = 2x$   
 $312.5 = x$

2) a)  $\log_{10} 100 = 2$

b)  $\log_{10} \frac{1}{10} = -1$

c)  $\log_{10} 1 = 0$

3)  $\log_a a = 1 \Rightarrow b \times \log_a a$

$b \times 1 = b \therefore$  always  
 $b$ .

1) a)  $x^2 = 9$

$x = \pm 3$

$\therefore x = 3$

b)  $4^1 = 3 - x$   
 $x = -1$

c)  $5^{x-3} = 0.04$

$5^{x-3} = \frac{1}{25}$

$5^{x-3} = 5^{-2}$

$x - 3 = -2$

$x = 1$

d)  $4^{2x-1} = 1$

$4^{2x-1} = 4^0$

$2x - 1 = 0$

$x = \frac{1}{2}$

e)  $x^{-3} = 0.125$

$x^{-3} = \frac{1}{8}$

$x^3 = 8$

$x = 2$

f)  $8^{x+7} = 2$

$(2^3)^{x+7} = 2^1$

$3x + 21 = 1$

$3x = -20$

$x = -\frac{20}{3}$

2)  $3^x = 25$

a)  $x \approx 2.9$

b)  $4^x = 14$

$x \approx 1.9$

c)  $2^{3.5} \approx 11.3$

d)  $10^x = 110$   
 $x \approx 2$

3) No!  $\log_8 0.5$

is a negative number  
 $\therefore$  he should have  
"reversed" the inequality  
as he divided by a negative  
number.

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**84** Laws of Logarithms

① a)  $\log(2 \times 8) = \log 16$

⑥  $\log(15 \div 5) = \log 3$

⑦  $\log\left(\frac{3 \times 8}{2}\right) = \log 12$

⑧  $\log\left(\frac{2}{5}\right) = \log 0.4$

⑨ a)  $\log x^2 + \log y^3$   
 $2 \log x + 3 \log y$  ✓

⑥  $\log x^5 - \log \sqrt{y}$   
 $\log x^5 - \log y^{\frac{1}{2}}$   
 $5 \log x - \frac{1}{2} \log y$  ✓

③ a)  $\log_2(3(x-1)) = 0$   
 $3(x-1) = 2^0$   
 $3(x-1) = 1$  ✓

⑥  $x-1 = \frac{1}{3}$   
 $x = \frac{4}{3}$  ✓

① a)  $\log_2 x^6 - \log_2 y$   
 $6 \log_2 x - \log_2 y$

⑥  $\log_2 x^7 + \log_2 z + \log_2 y^3$   
 $7 \log_2 x + \log_2 z + 3 \log_2 y$

⑦  $\log_2 8 + \log_2 x + \log_2 z^3$   
 $3 + \log_2 x + 3 \log_2 z$

②  $\log_2(5x-6)(3x+10) = 6$   
 $(5x-6)(3x+10) = 2^6$   
 $15x^2 + 32x - 60 = 64$   
 $15x^2 + 32x - 124 = 0$   
 $x = 2, x \neq -\frac{62}{15}$  ✓

③ a)  $\log_3(2x+1) + \log_3(x-1) = 5$   
 $\log_3(2x+1)(x-1) = 5$   
 $(2x+1)^2(x-1) = 3^5$   
 $(2x+1)^2(x-1) = 243$  ✓

⑥ Substitute in  $x=4$   
 $(8+1)^2(4-1)$   
 $9^2 \times (3) =$   
 $81 \times 3 = 243$  ✓

① a)  $\log_4(x-1)^2 - \log_4(x+3) = \frac{1}{2}$

$\log_4\left(\frac{(x-1)^2}{x+3}\right) = \frac{1}{2}$   
 $\frac{(x-1)^2}{x+3} = 4^{\frac{1}{2}}$   
 $(x-1)^2 = 2(x+3)$

$x^2 - 2x + 1 = 2x + 6$   
 $x^2 - 4x - 5 = 0$   
 $(x-5)(x+1) = 0$   
 $x = 5, x \neq -1$

⑥ Can't take the log of a negative

number for a real value  $\therefore x = -1$  is not a valid solution.

②  $4x+3 > 0$   
 $4x > -3$   
 $x > -\frac{3}{4}$

③ a)  $\log_8 2 + \log_2 x^4 + \log_2 y^{\frac{1}{2}}$   
 $\frac{1}{3} + 4 \log_2 x + \frac{1}{2} \log_2 y$   
 $\frac{1}{3} + 4p + \frac{1}{2}q$

④  $\log_8 x^9 - (\log_8 4 + \log_8 y^{\frac{1}{2}})$   
 $9 \log_8 x - \frac{2}{3} - \frac{1}{2} \log_8 y$   
 $9p - \frac{2}{3} - \frac{1}{2}q$

⑤  $\log x = \log_{10} x$   
 $\therefore \log 2 + \log x^2 - \log y^{\frac{1}{2}}$   
 $= \log\left(\frac{2x^2}{\sqrt{y}}\right)$  o.e.

# 85 Equations using Logarithms

① a)  $x = \log_3 13$   
 $x = 2.33$

b)  $x = \log_5 16$   
 $x = 1.72$

c)  $x = \log_2 0.91$   
 $x = -0.136$

② a)  $x+1 = \log_4 50$   
 $x = \log_4 50 - 1$   
 $x = 1.82$

b)  $1-x = \log_5 8$   
 $1 - \log_5 8 = x$   
 $x = -0.292$

c)  $2x+1 = \log_7 100$   
 $x = \frac{\log_7 100 - 1}{2}$   
 $x = 0.683$

③ a)  $\log 5^x = \log 2^{3x-1}$

$x \log 5 = (3x-1) \log 2$   
 $x \log 5 = 3x \log 2 - \log 2$

b)  $x \log 5 = 3x \log 2 - \log 2$   
 $\log 2 = 3x \log 2 - x \log 5$   
 $\log 2 = x(3 \log 2 - \log 5)$

$\frac{\log 2}{3 \log 2 - \log 5} = x$   
 $x = 1.47$

①  $\log 4^{x+1} = \log 3^{2x-1}$

$(x+1) \log 4 = (2x-1) \log 3$   
 $x \log 4 + \log 4 = 2x \log 3 - \log 3$   
 $\log 4 + \log 3 = 2x \log 3 - x \log 4$   
 $\log 4 + \log 3 = x(2 \log 3 - \log 4)$

$\frac{\log 4 + \log 3}{2 \log 3 - \log 4} = x$

$x = 3.06$

② a)  $(5^x)^2 + 2(5^x) - 8 = 0$

$p^2 + 2p - 8 = 0$

$(p+4)(p-2) = 0$

b)  $p = -4$  or  $p = 2$

c)  $5^x \neq -4$

$\therefore 5^x = 2$

$x = \log_5 2$

$x = 0.431$

③  $2^x = 3^{x-1}$

$\log 2^x = \log 3^{x-1}$

$x \log 2 = (x-1) \log 3$

$x \log 2 = x \log 3 - \log 3$

$\log 3 = x \log 3 - x \log 2$

$\log 3 = x(\log 3 - \log 2)$

$\frac{\log 3}{\log 3 - \log 2} = x$

$2.71 = x$

$\therefore y = 2^{7.09} \dots$

$y = 6.54$

$(2.71, 6.54)$

① a)  $\log 2^{3x-1} = \log 7 \times 5^{x-3}$

$\log 2^{3x-1} = \log 7 + \log 5^{x-3}$

$(3x-1) \log 2 = \log 7 + (x-3) \log 5$

$3x \log 2 - \log 2 = \log 7 + x \log 5 - 3 \log 5$

$x(3 \log 2 - \log 5) = \log 2 + \log 7 - 3 \log 5$

$x = \frac{\log 2 + \log 7 - 3 \log 5}{3 \log 2 - \log 5}$

$x = -4.66$

②  $6^{2x} - 6 \times 6^x + 8$

let  $6^x = p$  (if you want to)

$p^2 - 6p + 8 = 0$

$(p-2)(p-4) = 0$

$p = 2, p = 4$

$\therefore 6^x = 2$  or  $6^x = 4$

88 Continued

② continued

$$6^x = 2$$

$$x = \log_6 2$$

$$x = 0.387$$

or

$$6^x = 4$$

$$x = \log_6 4$$

$$x = 0.774$$

③  $5 \times 2 \times 2^{2x} + 2^2 \times 2^x + 1 = 0$

$$10 \times 2^{2x} + 4 \times 2^x + 1 = 0$$

let  $2^x = p$  (if you want)

$$10p^2 + 4p + 1 = 0$$

$b^2 - 4ac \geq 0$  for real roots to  $10p^2 + 4p + 1$

$$a = 10, b = 4, c = 1$$

$$4^2 - 4(10)(1) = -24$$

$\therefore$  No real roots.

N.B

you don't have to use

Substitutions such as

$p = 2^x$  and can just

factorise each 'o'

$$\text{eg: } 6^{2x} - 6 \times 6^{x+1} + 8$$

$$= (6^x - 4)(6^x - 2)$$

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