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A LEVEL MATHS

YEAR 1 PURE

ANSWERS

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76 Definite Integrals

① $I = \left[x^2 + 4x\right]_1^3$

$= (9 + 12) - (1 + 4)$

$= 21 - 4$

$= 16$ ✓

② ⓐ $I = \left[\frac{1}{5}x^5 + \frac{1}{2}x^2\right]_0^3$

$= \left(\frac{243}{5} + \frac{9}{2}\right) - (0 + 0)$

$= \frac{531}{10}$ o.e.

ⓑ $I = \left[-\frac{5}{x}\right]_1^4$

$= \left(-\frac{5}{4}\right) - \left(-\frac{5}{1}\right)$

$= \frac{15}{4}$

ⓒ $I = \left[2x^{\frac{1}{2}} - x\right]_4^9$

$= (6 - 9) - (4 - 4) = -3$

ⓓ $I = \left[\frac{2}{3}x^{3/2}\right]_1^{25}$

$= \frac{2}{3}\left[125 - 1\right]$

$= \frac{248}{3}$

③ $\int_1^8 (4 - 3t + t^{\frac{1}{3}})\,dt$

$= \left[4t - \frac{3t^2}{2} + \frac{3}{4}t^{\frac{4}{3}}\right]_1^8$

$= (32 - 96 + 12) - \left(4 - \frac{3}{2} + \frac{3}{4}\right)$

$= -\frac{221}{4}$

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① $\int 2x + x^{-\frac{1}{2}}$

$= \left[x^2 + 2x^{\frac{1}{2}}\right]_2^8$

$= \left(64 + 2(2\sqrt{2})\right) - \left(4 + 2\sqrt{2}\right)$

$(64 - 4) + (4\sqrt{2} - 2\sqrt{2})$

$60 + 2\sqrt{2}$ ✓

② $\int 6x^4 + x^2 - 2\,dx$

$I = \left[\frac{6}{5}x^5 + \frac{1}{3}x^3 - 2x\right]_1^3$

$= \left(\frac{1458}{5} + 9 - 6\right) - \left(\frac{6}{5} + \frac{1}{3} - 2\right)$

$= \frac{4426}{15}$

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①

$I = \left[y^2 + 4y\right]_n^{4n}$

$(16n^2 + 16n) - (n^2 + 4n) = 84$

$15n^2 + 12n = 84$

$5n^2 + 4n - 28 = 0$

$n = 2$ ✓ $n \neq -\frac{14}{5}$

$n = 2$

② $\int \left(\frac{1}{2}p^{-\frac{1}{2}} + \frac{3}{2}p^{\frac{1}{2}}\right)dp$

$I = \left[p^{\frac{1}{2}} + p^{\frac{3}{2}}\right]_3^{12}$

$= \left(\sqrt{12} + 12\sqrt{12}\right) - \left(\sqrt{3} + 3\sqrt{3}\right)$

$= 13\sqrt{12} - 4\sqrt{3}$

$= 13(2\sqrt{3}) - 4\sqrt{3}$

$= 52\sqrt{3} - 4\sqrt{3}$

$= 48\sqrt{3}$

77 Area Under Curves

① $y=0 \therefore 9-x^2=0$

$x=\pm 3$

ⓐ $(3,0)$ and $(-3,0)$.

ⓑ $A=\int_0^3 9-x^2\,dx$

$A=\left[9x-\frac{1}{3}x^3\right]_0^3$

$A=\left(9(3)-\frac{1}{3}(27)\right)-(0-0)$

$=27-9$

$=18$ ✓

② ⓐ $x(x^2-5x+6)$

$x(x-3)(x-2)$

ⓑ $A(0,0)$ $B(2,0)$ $C(3,0)$

ⓒ $A=\int_0^2 x^3-5x^2+6x\,dx$

$A=\left[\frac{1}{4}x^4-\frac{5}{3}x^3+3x^2\right]_0^2$

$=\left(\frac{16}{4}-\frac{40}{3}+12\right)-(0+0+0)$

$=\frac{8}{3}$

① ⓐ $y=0 \therefore 16-\frac{1}{x^2}=0$

$16=\frac{1}{x^2}$

$x^2=\frac{1}{16}$

$x=\pm\frac{1}{4} \therefore \left(\frac{1}{4},0\right)$

ⓑ $A=\int_{\frac{1}{4}}^4 16-x^{-2}\,dx$

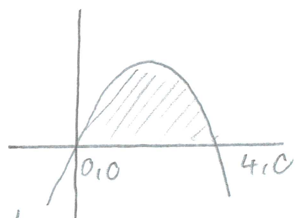
$A=\left[16x+\frac{1}{x}\right]_{\frac{1}{4}}^4$

$A=\left(64+\frac{1}{4}\right)-(4+4)$

$=\frac{257}{4}-8$

$=\frac{225}{4}$ ✓

② ⓐ



ⓑ $A=\int_0^4 x(4-x)\,dx$

$A=\int_0^4 4x-x^2\,dx$

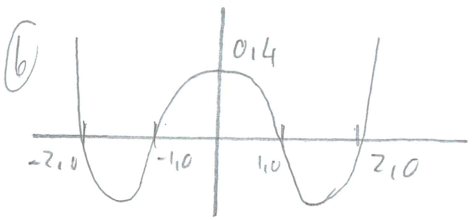
$A=\left[2x^2-\frac{1}{3}x^3\right]_0^4$

$A=\left(32-\frac{64}{3}\right)-(0+0)$

$A=\frac{32}{3}$

① ⓐ $(x+1)(x-1)(x+2)(x-2)$

ⓑ



ⓒ $A=\int_{-1}^1 x^4-5x^2+4\,dx$

$A=\left[\frac{1}{5}x^5-\frac{5}{3}x^3+4x\right]_{-1}^1$

$A=\left(\frac{1}{5}-\frac{5}{3}+4\right)-\left(-\frac{1}{5}-\frac{5}{3}-4\right)$

$=\frac{42}{5}$

② ⓐ $x^{\frac{1}{2}}-2=0$

$x^{\frac{1}{2}}=2$

$x=4 \therefore (4,0)$

ⓑ $1=a^{\frac{1}{2}}-2$

$3=a^{\frac{1}{2}}$

$9=a$

ⓒ $A=\int_4^9 \left(x^{\frac{1}{2}}-2\right)dx$

$=\left[\frac{2}{3}x^{3/2}-2x\right]_4^9$

$=\left(\frac{2}{3}(27)-2(9)\right)-\left(\frac{2}{3}(8)-8\right)$

$=(0)-\left(\frac{-8}{3}\right)$

$=\frac{8}{3}$

78 Areas Under Curves Yr1 Pure1

① $0 = x^2 - 4x$

ⓐ $0 = x(x-4)$

$\therefore$ $A(0,0)$ $B(4,0)$

ⓑ $\int x^2 - 4x \, dx = \frac{1}{3}x^3 - 2x^2 + c$

A to B

$\text{Area} = \left| \left[\frac{1}{3}x^3 - 2x^2 \right]_0^4 \right|$

$= \left| \left(\frac{64}{3} - 32 \right) - (0+0) \right|$

$= \frac{32}{3}$

B to x=6

$A = \left[\frac{1}{3}x^3 - 2x^2 \right]_4^6$

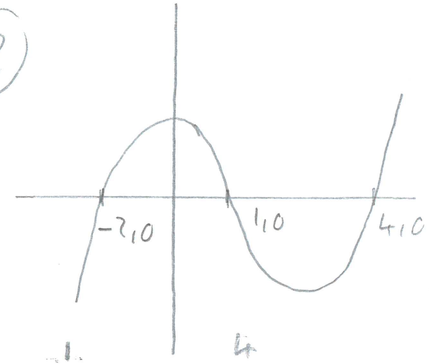
$= \left(\frac{216}{3} - 72 \right) - \left(-\frac{32}{3} \right)$

$= 0 + \frac{32}{3}$

$= \frac{32}{3}$

$\therefore \frac{32}{3} + \frac{32}{3} = 64/3$

②



$A = \int_{-2}^{1} + \left| \int_1^4 \right|$

$\int x^3 - 3x^2 - 6x + 8 \, dx =$

$\frac{1}{4}x^4 - x^3 - 3x^2 + 8x + c$

From -2 to 1

$A = \left[\frac{1}{4}x^4 - x^3 - 3x^2 + 8x \right]_{-2}^1$

$A = \left(\frac{1}{4} - 1 - 3 + 8 \right) - (4 + 8 - 12 - 16)$

$= \frac{17}{4} - -16$

$= \frac{81}{4}$

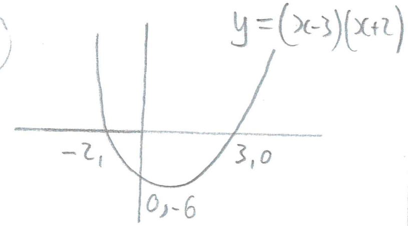
From 1 to 4

$A = \left| (64 - 64 - 48 + 32) - \left(\frac{17}{4} \right) \right|$

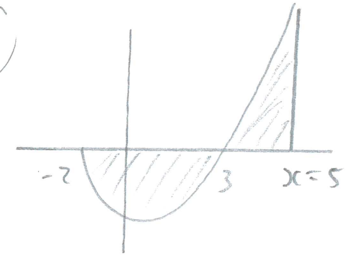
$= \frac{81}{4}$

$\therefore \frac{81}{4} + \frac{81}{4} = \frac{81}{2}$

1 ⓐ



ⓑ



$A = \left| \int_{-2}^{3} x^2 - x - 6 \, dx \right| + \int_3^5 x^2 - x - 6 \, dx$

$I = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x$

From -2 to 3

$A = \left| \left[\frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x \right]_{-2}^3 \right|$

$= \left| \left(9 - \frac{9}{2} - 18 \right) - \left(-\frac{8}{3} - 2 + 12 \right) \right|$

$= \frac{125}{6}$

From 3 to 5

$A = \left[\frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x \right]_3^5$

$= \left(\frac{125}{3} - \frac{25}{2} - 30 \right) - \left(9 - \frac{9}{2} - 18 \right)$

$= \frac{38}{3}$

$\therefore \frac{125}{6} + \frac{38}{3} = \frac{67}{2}$

② ⓐ $(x-1)(x-2)(x-5)$

$(x-1)\left[x^2 - 7x + 10 \right]$

$x^3 - 7x^2 + 10x$

$\quad - x^2 + 7x - 10$

$x^3 - 8x^2 + 17x - 10$ ✓

ⓑ $A = \int_1^2 + \left| \int_2^5 \right|$

$\int x^3 - 8x^2 + 17x - 10 \, dx =$

$\frac{1}{4}x^4 - \frac{8}{3}x^3 + \frac{17}{2}x^2 - 10x$

18 Continued

9b continued.

From 1 to 2

$$A = \left[\frac{1}{4}x^4 - \frac{8}{3}x^3 + \frac{17}{2}x^2 - 10x\right]_1^2$$

$$= \left(4 - \frac{64}{3} + 34 - 20\right) - \left(\frac{1}{4} - \frac{8}{3} + \frac{17}{2} - 10\right)$$

$$\left(-\frac{10}{3}\right) - \left(-\frac{47}{12}\right) = \frac{7}{12}$$

From 2 to 5

$$A = \left| \left[\frac{1}{4}x^4 - \frac{8}{3}x^3 + \frac{17}{2}x^2 - 10x\right]_2^5 \right|$$

$$= \left| \left(\frac{625}{4} - \frac{1000}{3} + \frac{425}{2} - 50\right) - \left(-\frac{10}{3}\right) \right|$$

$$= \frac{45}{4}$$

$$\text{Total} = \frac{7}{12} + \frac{45}{4} = \frac{71}{6}$$

1 a) $x^{3/2} - 8 = 0$

$x^{3/2} = 8$

$x = 8^{2/3}$

$x = 4$

$\therefore A(4,0)$

b) $19 = b^{3/2} - 8$

$27 = b^{3/2}$

$27^{2/3} = b$

$9 = b$

$B(9,0)$

c) $A = \left|\int_0^4\right| + \int_4^9$

$$\int x^{3/2} - 8\,dx = \frac{2}{5}x^{5/2} - 8x + c$$

From 0 to 4

$$A = \left| \left[\frac{2}{5}x^{5/2} - 8x\right]_0^4 \right|$$

$$= \left| \left(\frac{64}{5} - 32\right) - (0 - 0) \right|$$

$$= \frac{96}{5}$$

From 4 to 9

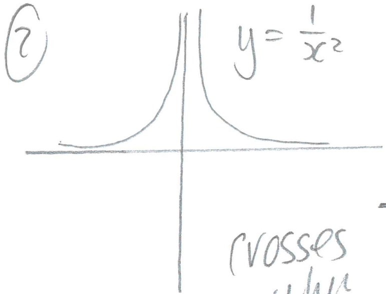
$$A = \left[\frac{2}{5}x^{5/2} - 8x\right]_4^9$$

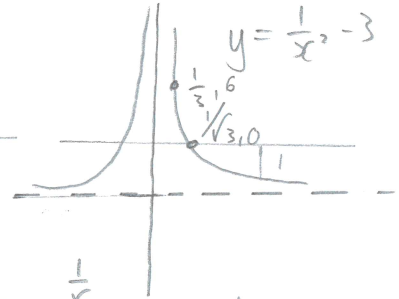
$$= \left(\frac{486}{5} - 72\right) - \left(-\frac{96}{5}\right)$$

$$= \frac{222}{5}$$

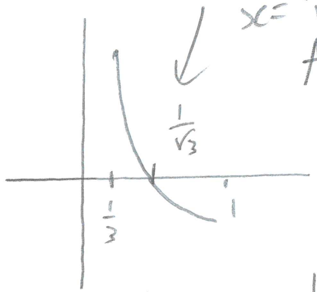
$$\text{Total} = \frac{96}{5} + \frac{222}{5} = \frac{318}{5}$$

2)





crosses when $x = \frac{1}{\sqrt{3}}$



$$A = \int_{\frac{1}{3}}^{\frac{1}{\sqrt{3}}} + \left| \int_{\frac{1}{\sqrt{3}}}^{1} \right|$$

$8 - 4\sqrt{3}$

$$\int x^{-2} - 3\,dx = -\frac{1}{x} - 3x + c$$

From $\frac{1}{3}$ to $\frac{1}{\sqrt{3}}$

$$A = \left[-\frac{1}{x} - 3x\right]_{\frac{1}{3}}^{1/\sqrt{3}}$$

$$= (-\sqrt{3} - \sqrt{3}) - (-3 - 1)$$

From $\frac{1}{\sqrt{3}}$ + 1

$$A = \left| \left[-\frac{1}{x} - 3x\right]_{\frac{1}{\sqrt{3}}}^{1} \right|$$

$$= \left| (-1 - 3) - (-\sqrt{3} - \sqrt{3}) \right|$$

Total $(4 - 2\sqrt{3}) + 4 + 2\sqrt{3})$

79 Areas Between Curves and Lines

1 (a) $y = x^2$
$y = 2x$

$\therefore x^2 = 2x$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, x = 2$

when $x = 0, y = 0$
when $x = 2$ $y = 4$

$A(0,0)$ $B(2,0)$

(b) N.B You can do line minus curve or Area under curve and triangle

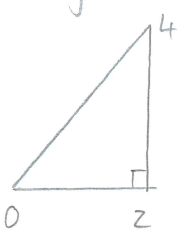
Line minus Curve Method

$$A = \int_0^2 2x - (x^2)\,dx$$

$$A = \left[x^2 - \tfrac{1}{3}x^3\right]_0^2$$

$$= \left(4 - \tfrac{8}{3}\right) - (0+0) = \tfrac{4}{3}$$

Triangle Method.



$A = \frac{4 \times 2}{2}$

$= 4$

$$\int_0^2 x^2\,dx = \left[\tfrac{1}{3}x^3\right]_0^2$$

$$= \tfrac{8}{3}$$

$$4 - \tfrac{8}{3} = \tfrac{4}{3} \checkmark$$

1 Curve minus Line.

$$\int_0^2 (-x^3+8) - (8-4x)\,dx$$

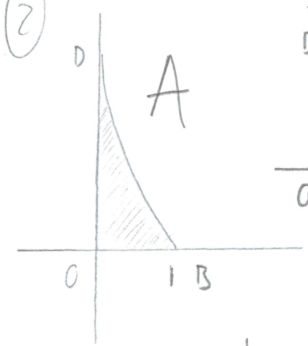
$$\int_0^2 -x^3 + 4x\,dx$$

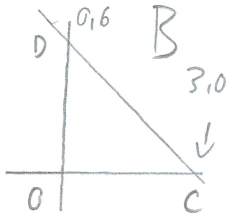
$$\left[-\tfrac{1}{4}x^4 + 2x^2\right]_0^2$$

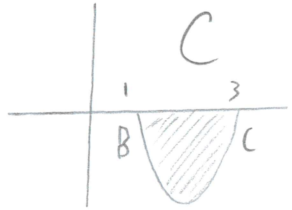
$$= (-4+8) - (0+0)$$

$$= 4 \checkmark$$

2







$y = x^3 - 2x^2 - 5x + 6$

~~Area~~ $= \int x^3 - 2x^2 - 5x + 6\,dx$

$= \left[\tfrac{1}{4}x^4 - \tfrac{2}{3}x^3 - \tfrac{5}{2}x^2 + 6x\right]$

(A)

$$\text{Area} = \left[\tfrac{1}{4}x^4 - \tfrac{2}{3}x^3 - \tfrac{5}{2}x^2 + 6x\right]_0^1$$

$$= \left(\tfrac{1}{4} - \tfrac{2}{3} - \tfrac{5}{2} + 6\right) - (0+0+0+0)$$

$$= \tfrac{37}{12}$$

(B) $\tfrac{1}{2} \times 3 \times 6 = 9$

(B) − (A) $= 9 - \tfrac{37}{12}$

$= \tfrac{71}{12}$

(C)

$$\text{Area} = \left|\left[\tfrac{1}{4}x^4 - \tfrac{2}{3}x^3 - \tfrac{5}{2}x^2 + 6x\right]_1^3\right|$$

$$= \left|\left(\tfrac{81}{4} - 18 - \tfrac{45}{2} + 18\right) - \left(\tfrac{1}{4} - \tfrac{2}{3} - \tfrac{5}{2} + 6\right)\right|$$

$$= \tfrac{16}{3}$$

$$\tfrac{71}{12} + \tfrac{16}{3} = \tfrac{45}{4} \checkmark$$

1 (a) Gradient = 1 of tangent

$\therefore \frac{dy}{dx} = 1$ @ P

$y = 4x^{\frac{1}{2}}$

$\frac{dy}{dx} = 2x^{-\frac{1}{2}}$

or $\frac{2}{\sqrt{x}}$

$1 = \frac{2}{\sqrt{x}}$

$\sqrt{x} = \frac{2}{1}$

$x = 4$

when $x = 4$, $y = 4\sqrt{4}$
$y = 8$

$P(4,8)$

79 Continued

1 (b) $y = x + a$

$8 = 4 + a$

$a = 4$

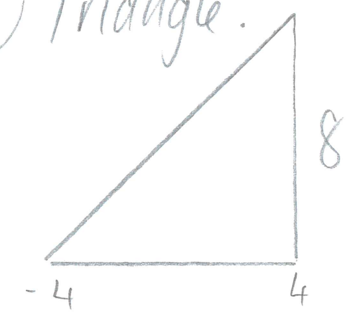
$\therefore y = x + 4$

$0 = x + 4$

$-4 = x$

$\therefore A(-4, 0)$

(c) Triangle.



$\text{Area} = \frac{8 \times 8}{}$

$= 32$

Area under curve from 0 to 4

$A = \int_0^4 4x^{\frac{1}{2}} dx$

$A = \left[\frac{8}{3} x^{3/2} \right]_0^4$

$A = \left(\frac{8}{3}(8) - 0 \right)$

$A = \frac{64}{3}$

Shaded area = Triangle − Area under curve

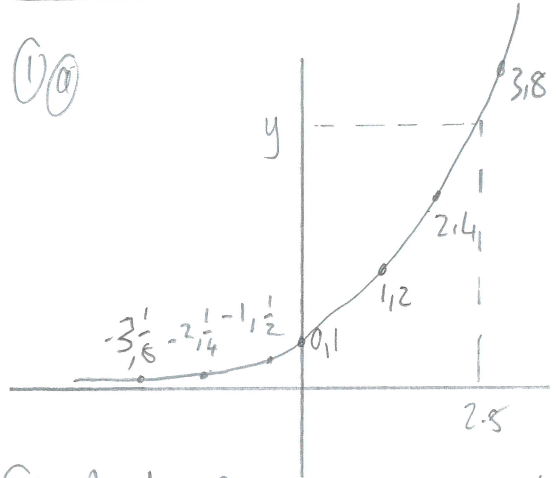
$\therefore 32 - \frac{64}{3}$

$= \frac{32}{3}$

Year 1 6C Exponential Functions

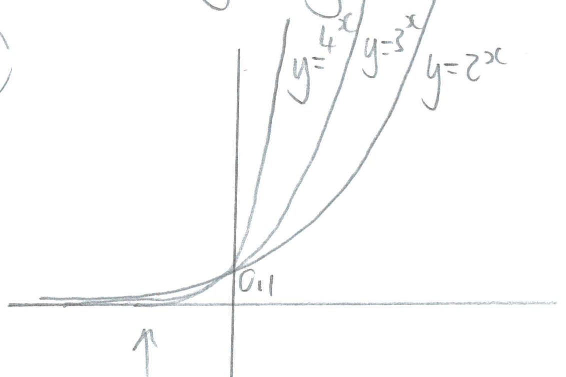
← 4,16

1 (a)



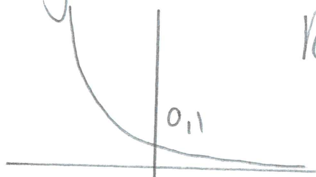
(b) Reads from graph as shown above giving ≈ 5.5 ish!

2



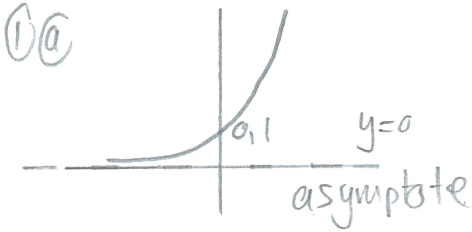
↑ hard to see but $2^x > 3^x > 4^x$ for negative values

3

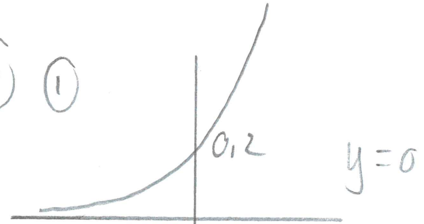


Reflection of $y=2^x$ in y axis

1 (a)

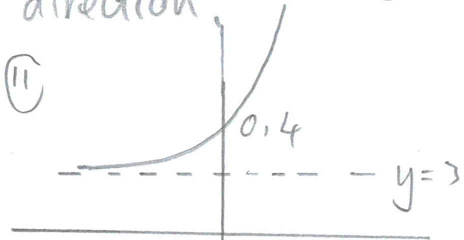


(b) (i)



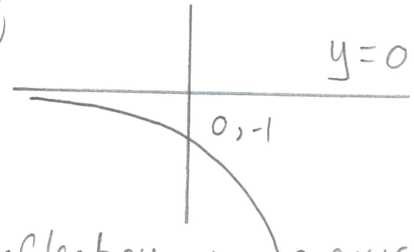
SF 2 Stretch in +y direction.

(ii)



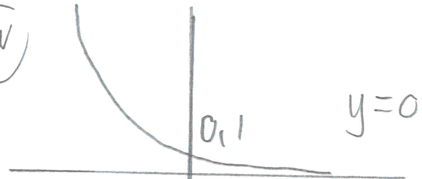
translation $\binom{0}{3}$

(iii)



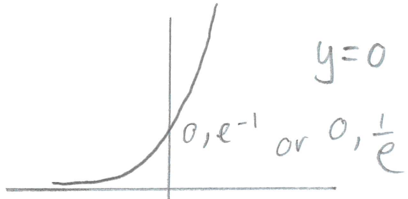
Reflection in x axis

(iv)



Reflection in y axis.

(v)



Translation $\binom{1}{0}$

2 Substitute coordinates into equation.

when $x=2, y=18$

$\therefore 18 = pa^2$

when $x=3, y=54$

$\therefore 54 = pa^3$

(b) $54 = pa^3$ (1)

$18 = pa^2$ (2)

(1) ÷ (2)

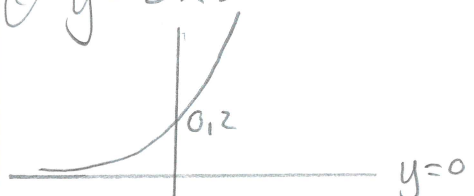
$3 = a$

Sub back in to (2)

$18 = p(3)^2$

$p = 2$

(c) $y = 2 \times 3^x$



(SF 2 in +y direction Stretch)

(i)

when $x=0$, $y = 3 \times \left(\frac{1}{2}\right)^{-1}$

$y = 6$

asymptote: $y=0$

2 Sub in points

when $x=-1$ $y=0$

$\therefore 0 = ab^{-1} + 2$

or $\frac{a}{b} = -2$

or $a = -2b$ (1)

when $y=-2$ $x=0$

$\therefore -2 = ab^0 + 2$

$-2 = a + 2$

$-4 = a$ (2)

If $a=-4, b=2$

Equation: $y = -4 \times 2^x + 2$

At the point $(3, p)$

$p = -4 \times 2^3 + 2$

$= -32 + 2$

$= -30$ ✓

80 Continued.

(3) $y = ab^x + c$

Crosses y axis when

$x = 0$ so $y = ab^0 + c$

$y = a(1) + c$

$y = a + c$

$\therefore (0, a + c)$

Steve Blades