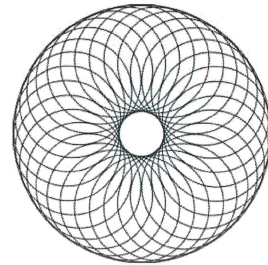


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YEAR 1 PURE**



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- (1) Indices
- (2) Expanding Brackets
- (3) Factorising Expressions
- (4) More Indices (Negative and Fractional)
- (5) Working with Surds
- (6) Solving Quadratic Equations
- (7) Completing the Square for Quadratics Expressions
- (8) Function Notation
- (9) Sketching Quadratic Graphs
- (10) The Discriminant for Quadratic Equations
- (11) Applications of Quadratics Equations
- (12) Solving Linear Simultaneous Equations
- (13) Linear & Non-Linear Simultaneous Equations
- (14) Graphing Simultaneous Equations
- (15) Linear Inequalities
- (16) Quadratic Inequalities
- (17) Graphing Inequalities
- (18) Shading Inequalities
- (19) Cubic Graphs
- (20) Quartic Graphs
- (21) Reciprocal Graphs
- (22) The Intersection of Graphs
- (23) Transforming Graphs (Translations)
- (24) Transforming Graphs (Stretching/Reflecting)
- (25) Straight Line Graphs in the form $y = mx + c$
- (26) More Straight Line Graphs
- (27) Straight Line Graphs (Parallel & Perpendicular)
- (28) The Geometry of Straight Lines
- (29) The Application of Linear Graphs
- (30) Circle Geometry Midpoint & Perpendicular

- (31) The Equation of a Circle
- (32) Circles and Straight Lines (Intersections)
- (33) Circles (Tangents and Chords)
- (34) Circles and Triangles
- (35) Algebraic Fractions
- (36) Polynomial Division
- (37) The Factor and Remainder Theorem
- (38) An Introduction to Mathematical Proof
- (39) Methods of Proof
- (40) Binomial Expansion (Using Pascal's Triangle)
- (41) Binomial Expansion (Factorial Notation)
- (42) Binomial Expansion (The $\binom{n}{r}$ Method)
- (43) Binomial Expansion (Problem Solving)
- (44) Binomial Expansion (Estimations and Approximations)
- (45) The Cosine Rule
- (46) The Sine Rule
- (47) Areas of a Triangles
- (48) Triangles (Problem Solving)
- (49) Sine, Cosine & Tangent Graphs
- (50) Transforming Graphs (Trigonometry)
- (51) The 'CAST' Diagram for Trig Ratios
- (52) Trigonometry (Exact Values)
- (53) Proving Trigonometric Identities
- (54) Solving Basic Trigonometric Equations
- (55) More Challenging Trigonometric Equations
- (56) Using Identities to Solve Trig Equations
- (57) Vectors (Introduction)

- (58) Vector Notation (Column and i and j form)
- (59) Vectors (Magnitude and Direction)
- (60) Vectors (Position and Direction Vectors)
- (61) Vector Geometry
- (62) Application of Vectors
- (63) Differentiation (Gradients of Curves)
- (64) Differentiation from 1st Principles
- (65) Differentiating x^n (Basic Powers of)
- (66) Differentiation (Quadratic Expression)
- (67) Differentiation (Multiple Terms)
- (68) Differentiation (Gradients, Tangents and Normals)
- (69) Differentiation (Increasing and Decreasing Functions)
- (70) Differentiation (Stationary Points)
- (71) Differentiation (Gradient Functions)
- (72) The Applications of Differentiation
- (73) Integration (Basic Expressions (x^n))
- (74) Indefinite Integrals
- (75) Integration (Finding c and Finding Functions)
- (76) Integration (Definite Integrals)
- (77) Integration (Basic Areas Under Curves)
- (78) Integration ('Negative and Positive Areas')
- (79) Integration (Areas between Curves and Lines)
- (80) Basic Exponential Functions
- (81) 'The' Exponential Function $y = e^x$
- (82) Applications of Basic Exponential Models
- (83) Logarithms (Simplifying & Evaluating)
- (84) Logarithms (The Log Laws)
- (85) Logarithms (Log and Exponential Equations)

76 Definite Integrals

$$\begin{aligned} \textcircled{1} I &= [x^2 + 4x]^3_1 \\ &= (9 + 12) - (1 + 4) \\ &= 21 - 4 \\ &= 16 \checkmark \end{aligned}$$

$$\begin{aligned} \textcircled{2} \textcircled{a} I &= \left[\frac{1}{5}x^5 + \frac{1}{2}x^2 \right]_0^3 \\ &= \left(\frac{243}{5} + \frac{9}{2} \right) - (0 + 0) \\ &= \frac{531}{10} \text{ a.e.} \end{aligned}$$

$$\begin{aligned} \textcircled{6} I &= \left[-\frac{5}{x} \right]_{-1}^4 \\ &= \left(-\frac{5}{4} \right) - \left(-\frac{5}{1} \right) \\ &= \frac{15}{4} \end{aligned}$$

$$\begin{aligned} \textcircled{c} I &= \left[2x^{\frac{1}{2}} - x \right]_4^9 \\ &= (6 - 9) - (4 - 4) = -3 \end{aligned}$$

$$\begin{aligned} \textcircled{d} I &= \left[\frac{2}{3}x^{\frac{3}{2}} \right]_1^{25} \\ &= \frac{2}{3} [125 - 1] \\ &= \frac{248}{3} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \int_1^8 (4 - 3t + t^{\frac{1}{3}}) dt \\ &= \left[4t - \frac{3t^2}{2} + \frac{3}{4}t^{\frac{4}{3}} \right]_1^8 \\ &= \left(32 - 96 + 12 \right) \\ &\quad - \left(4 - \frac{3}{2} + \frac{3}{4} \right) \\ &= -\frac{221}{4} \end{aligned}$$

$$\begin{aligned} \textcircled{1} \int_2^8 (2x + x^{-\frac{1}{2}}) dx \\ &= \left[x^2 + 2x^{\frac{1}{2}} \right]_2^8 \\ &= (64 + 2(2\sqrt{2})) - \\ &\quad \left(4 + 2\sqrt{2} \right) \\ &= (64 - 4) + (4\sqrt{2} - 2\sqrt{2}) \\ &= 60 + 2\sqrt{2} \checkmark \end{aligned}$$

$$\begin{aligned} \textcircled{2} \int_1^3 (6x^4 + x^2 - 2) dx \\ I &= \left[\frac{6}{5}x^5 + \frac{1}{3}x^3 - 2x \right]_1^3 \\ &= \left(\frac{1458}{5} + 9 - 6 \right) - \left(\frac{6}{5} + \frac{1}{3} - 2 \right) \\ &= \frac{4426}{15} \end{aligned}$$

$$\begin{aligned} &= 13(2\sqrt{3}) - 4\sqrt{3} \\ &= 52\sqrt{3} - 4\sqrt{3} \\ &= \underline{48\sqrt{3}} \end{aligned}$$

$$\begin{aligned} \textcircled{1} I &= [y^2 + 4y]_n^{4n} \\ (16n^2 + 16n) - (n^2 + 4n) &= 84 \\ 15n^2 + 12n &= 84 \\ 5n^2 + 4n - 28 &= 0 \\ n = 2 \quad n \neq -\frac{14}{5} \end{aligned}$$

$n = 2$

$$\begin{aligned} \textcircled{2} \int_3^{12} \left(\frac{1}{2}p^{-\frac{1}{2}} + \frac{3}{2}p^{\frac{1}{2}} \right) dp \\ I &= \left[p^{\frac{1}{2}} + p^{\frac{3}{2}} \right]_3^{12} \\ &= (\sqrt{12} + 12\sqrt{2}) - (\sqrt{3} + 3\sqrt{3}) \\ &= 13\sqrt{12} - 4\sqrt{3} \end{aligned}$$

77 Areas Under Curves

① $y=0 \therefore 9-x^2=0$
 $x=\pm 3$

② (3,0) and (-3,0)

③ $A = \int_0^3 9-x^2 dx$

$A = \left[9x - \frac{1}{3}x^3 \right]_0^3$

$A = \left(9(3) - \frac{1}{3}(27) \right) - (0-0)$
 $= 27-9$
 $= 18 \checkmark$

④ ① $x(x^2-5x+6)$
 $x(x-3)(x-2)$

② A(0,0) B(2,0) C(3,0)

③ $A = \int_0^3 x^3 - 5x^2 + 6x dx$

$A = \left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + 3x^2 \right]_0^3$

$= \left(\frac{16}{4} - \frac{40}{3} + 12 \right) - (0+0+0)$

$= \frac{8}{3}$

① ① $y=0 \therefore 16-\frac{1}{x^2}=0$

$16 = \frac{1}{x^2}$

$x^2 = \frac{1}{16}$

$x = \pm \frac{1}{4} \therefore \left(\frac{1}{4}, 0 \right)$

② $A = \int_{\frac{1}{4}}^4 16-x^{-2} dx$

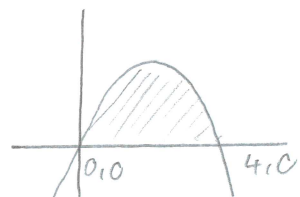
$A = \left[16x + \frac{1}{x} \right]_{\frac{1}{4}}^4$

$A = \left(64 + \frac{1}{4} \right) - \left(4 + 4 \right)$

$= \frac{257}{4} - 8$

$= \frac{225}{4} \checkmark$

③ ①



② $A = \int_0^4 x(4-x) dx$

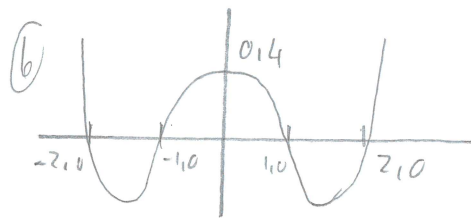
$A = \int_0^4 4x - x^2 dx$

$A = \left[2x^2 - \frac{1}{3}x^3 \right]_0^4$

$A = \left(32 - \frac{64}{3} \right) - (0+0)$

$A = \frac{32}{3}$

④ ① $(x+1)(x-1)(x+2)(x-2)$



② $A = \int_{-1}^1 x^4 - 5x^2 + 4 dx$

$A = \left[\frac{1}{5}x^5 - \frac{5}{3}x^3 + 4x \right]_{-1}^1$

$A = \left(\frac{1}{5} - \frac{5}{3} + 4 \right) - \left(-\frac{1}{5} - \frac{5}{3} - 4 \right)$

$= \frac{42}{5}$

⑤ ① $x^{\frac{1}{2}} - 2 = 0$

$x^{\frac{1}{2}} = 2$

$x = 4 \therefore (4, 0)$

② $1 = a^{\frac{1}{2}} - 2$

$3 = a^{\frac{1}{2}}$

$9 = a$

③ $A = \int_4^9 (x^{\frac{1}{2}} - 2) dx$

$= \left[\frac{2}{3}x^{\frac{3}{2}} - 2x \right]_4^9$

$= \left(\frac{2}{3}(27) - 2(9) \right) - \left(\frac{2}{3}(8) - 8 \right)$

$= (0) - \left(-\frac{8}{3} \right)$

$= \frac{8}{3}$

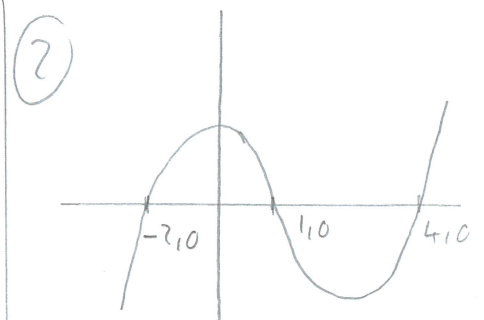
78 Areas Under Curves Yel Purel

① $0 = x^2 - 4x$
 ② $0 = x(x-4)$
 $\therefore A(0,0) B(4,0)$

③ $\int x^2 - 4x dx = \frac{1}{3}x^3 - 2x^2 + c$

A to B
 Area = $\left| \left[\frac{1}{3}x^3 - 2x^2 \right]_0^4 \right|$
 $= \left| \left(\frac{64}{3} - 32 \right) - (0+0) \right|$
 $= \frac{32}{3}$

B to x=6
 $A = \left[\frac{1}{3}x^3 - 2x^2 \right]_4^6$
 $= \left(\frac{216}{3} - 72 \right) - \left(-\frac{32}{3} \right)$
 $= 0 + \frac{32}{3}$
 $= \frac{32}{3}$
 $\therefore \frac{32}{3} + \frac{32}{3} = 64/3$



$A = \int_{-2}^1 + \left| \int_1^4 \right|$

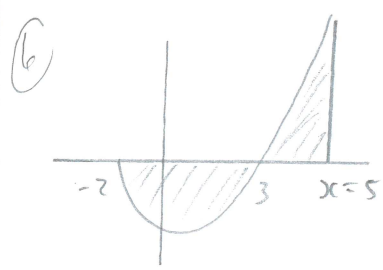
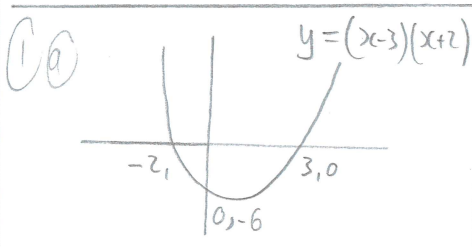
$\int x^3 - 3x^2 - 6x + 8 dx =$
 $\frac{1}{4}x^4 - x^3 - 3x^2 + 8x + c$

From -2 to 1
 $A = \left[\frac{1}{4}x^4 - x^3 - 3x^2 + 8x \right]_{-2}^1$

$A = \left(\frac{1}{4} - 1 - 3 + 8 \right) - \left(4 + 8 - 12 - 16 \right)$
 $= \frac{17}{4} - -16$
 $= \frac{81}{4}$

From 1 to 4
 $A = \left(64 - 64 - 48 + 32 \right) - \left(\frac{17}{4} \right)$

$= \frac{81}{4}$
 $\therefore \frac{81}{4} + \frac{81}{4} = \frac{81}{2}$



$A = \left| \int_{-2}^3 x^2 - x - 6 dx \right| + \int_3^5 x^2 - x - 6 dx$
 $I = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x$

From -2 to 3
 $A = \left| \left[\frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x \right]_{-2}^3 \right|$

$= \left| \left(9 - \frac{9}{2} - 18 \right) - \left(-\frac{8}{3} - 2 + 12 \right) \right|$
 $= \frac{125}{6}$

From 3 to 5
 $A = \left[\frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x \right]_3^5$
 $= \left(\frac{125}{3} - \frac{25}{2} - 30 \right) - \left(9 - \frac{9}{2} - 18 \right)$
 $= \frac{38}{3}$

$\therefore \frac{125}{6} + \frac{38}{3} = \frac{67}{2}$

⑥ $(x-1)(x-2)(x-5)$
 $(x-1)[x^2 - 7x + 10]$
 $x^3 - 7x^2 + 10x - x^2 + 7x - 10$
 $x^3 - 8x^2 + 17x - 10$ ✓

⑦ $A = \int_1^2 + \left| \int_2^5 \right|$
 $\int x^3 - 8x^2 + 17x - 10 dx =$
 $\frac{1}{4}x^4 - \frac{8}{3}x^3 + \frac{17}{2}x^2 - 10x$

78 Continued

(b) continued.

From 1 to 2

$$A = \left[\frac{1}{4}x^4 - \frac{8}{3}x^3 + \frac{17}{2}x^2 - 10x \right]_1^2$$

$$= \left(4 - \frac{64}{3} + 34 - 20 \right) - \left(\frac{1}{4} - \frac{8}{3} + \frac{17}{2} - 10 \right)$$

$$\left(-\frac{10}{3} \right) - \left(-\frac{47}{12} \right) = \frac{7}{12}$$

From 2 to 5

$$A = \left| \left[\frac{1}{4}x^4 - \frac{8}{3}x^3 + \frac{17}{2}x^2 - 10x \right]_2^5 \right|$$

$$= \left| \left(\frac{625}{4} - \frac{1000}{3} + \frac{425}{2} - 50 \right) - \left(-\frac{10}{3} \right) \right|$$

$$= \frac{45}{4}$$

Total = $\frac{7}{12} + \frac{45}{4}$

$$= \frac{71}{6}$$

①(a) $x^{3/2} - 8 = 0$

$$x^{3/2} = 8$$

$$x = 8^{2/3}$$

$$x = 4$$

$\therefore A(4, 0)$

⑥ $19 = b^{3/2} - 8$

$$27 = b^{3/2}$$

$$27^{2/3} = b$$

$$9 = b$$

$B(9, 0)$

⑦ $A = \left| \int_0^4 \right| + \left| \int_4^9 \right|$

$$\int x^{3/2} - 8 dx = \frac{2}{5}x^{5/2} - 8x + c$$

From 0 to 4

$$A = \left| \left[\frac{2}{5}x^{5/2} - 8x \right]_0^4 \right|$$

$$= \left| \left(\frac{64}{5} - 32 \right) - (0 - 0) \right|$$

$$= \frac{96}{5}$$

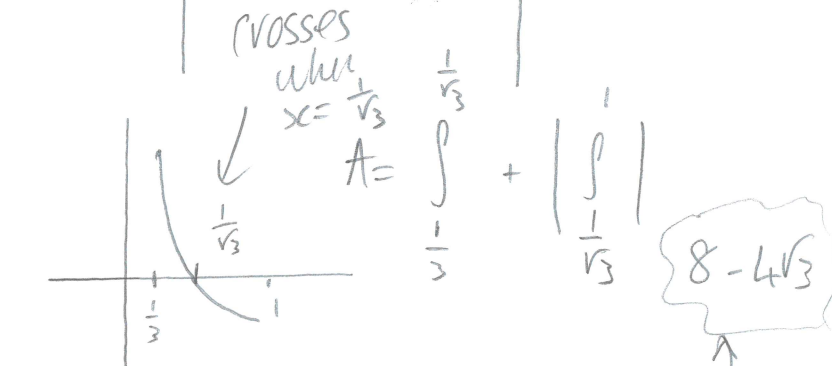
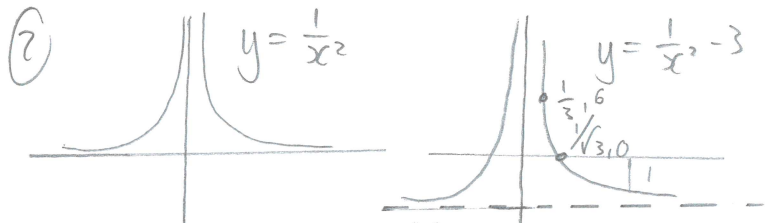
From 4 to 9

$$A = \left[\frac{2}{5}x^{5/2} - 8x \right]_4^9$$

$$= \left(\frac{486}{5} - 72 \right) - \left(-\frac{96}{5} \right)$$

$$= \frac{222}{5}$$

Total = $\frac{96}{5} + \frac{222}{5} = \frac{318}{5}$



$$\int x^2 - 3 dx = -\frac{1}{x} - 3x + c$$

From 1/3 to 1/sqrt(3)

$$A = \left[-\frac{1}{x} - 3x \right]_{1/3}^{1/\sqrt{3}}$$

$$= (-\sqrt{3} - \sqrt{3}) - (-3 - 1)$$

From 1/sqrt(3) to 1

$$A = \left[-\frac{1}{x} - 3x \right]_{1/\sqrt{3}}^1$$

$$= (-1 - 3) - (-\sqrt{3} - \sqrt{3})$$

Total = $(4 - 2\sqrt{3}) + 4 + 2\sqrt{3}$

79 Areas Between Curves and Lines

① (a) $y = x^2$
 $y = 2x$

∴ $x^2 = 2x$
 $x^2 - 2x = 0$
 $x(x-2) = 0$
 $x = 0, x = 2$

when $x = 0, y = 0$
when $x = 2, y = 4$

A(0,0) B(2,0)

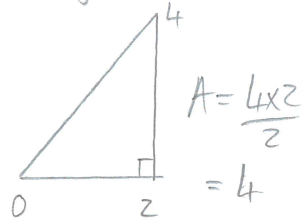
⑥ N.B You could line minus curve or Area under curve and triangle
Line minus Curve Method

$$A = \int_0^2 2x - (x^2) dx$$

$$A = \left[x^2 - \frac{1}{3}x^3 \right]_0^2$$

$$= \left(4 - \frac{8}{3} \right) - (0+0) = \frac{4}{3}$$

Triangle Method.



$$\int_0^2 x^2 dx = \left[\frac{1}{3}x^3 \right]_0^2$$

$$= \frac{8}{3}$$

$$4 - \frac{8}{3} = \frac{4}{3} \checkmark$$

① Curve minus Line.

$$\int_0^2 (-x^3 + 8) - (8 - 4x) dx$$

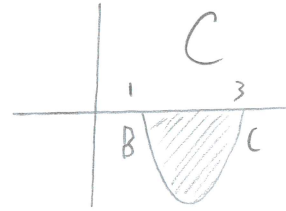
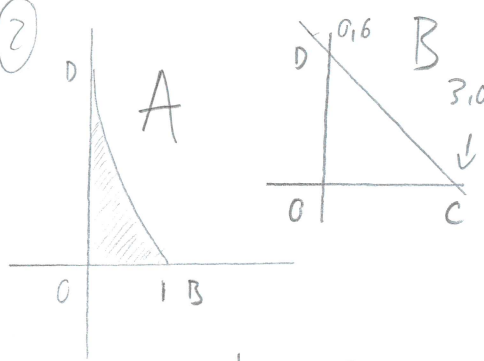
$$\int_0^2 -x^3 + 4x dx$$

$$\left[-\frac{1}{4}x^4 + 2x^2 \right]_0^2$$

$$= (-4 + 8) - (0 + 0)$$

$$= 4 \checkmark$$

②



$$y = x^3 - 2x^2 - 5x + 6$$

$$\text{Area} = \int x^3 - 2x^2 - 5x + 6 dx$$

$$= \left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{5}{2}x^2 + 6x \right]$$

(A)

$$\text{Area} = \left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{5}{2}x^2 + 6x \right]_0^1$$

$$= \left(\frac{1}{4} - \frac{2}{3} - \frac{5}{2} + 6 \right) - (0+0+0+0)$$

$$= \frac{37}{12}$$

(B) $\frac{1}{2} \times 3 \times 6 = 9$

(B) - (A) = $9 - \frac{37}{12}$

$$= \frac{71}{12}$$

$$\text{Area} = \left| \left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{5}{2}x^2 + 6x \right]_0^3 \right|$$

$$= \left| \left(\frac{81}{4} - 18 - \frac{45}{2} + 18 \right) - \left(\frac{1}{4} - \frac{2}{3} - \frac{5}{2} + 6 \right) \right|$$

$$= \frac{16}{3}$$

$$\frac{71}{12} + \frac{16}{3} = \frac{45}{4} \checkmark$$

① (a) Gradient = 1 of tangent

∴ $\frac{dy}{dx} = 1$ @ P

$$y = 4x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = 2x^{-\frac{1}{2}}$$

$$\text{or } \frac{2}{\sqrt{x}}$$

$$1 = \frac{2}{\sqrt{x}}$$

$$\sqrt{x} = \frac{2}{1}$$

$$x = 4$$

When $x = 4, y = 4\sqrt{4}$
 $y = 8$

P(4, 8)

79) Continued

①⑥ $y = x + a$

$$8 = 4 + a$$

$$a = 4$$

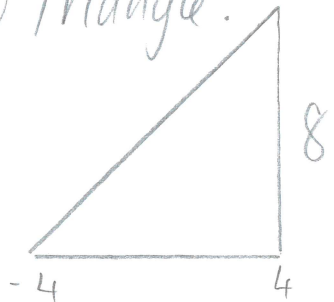
$$\therefore y = x + 4$$

$$0 = x + 4$$

$$-4 = x$$

$$\therefore A(-4, 0)$$

⑦ Triangle.



$$\text{Area} = \frac{8 \times 8}{2} = 32$$

Area under curve from $x = 0$ to $x = 4$

$$A = \int_0^4 4x^{\frac{1}{2}} dx$$

$$A = \left[\frac{8}{3} x^{\frac{3}{2}} \right]_0^4$$

$$A = \left(\frac{8}{3} (8) - 0 \right)$$

$$A = \frac{64}{3}$$

Shaded area =
Triangle - Area
under curve

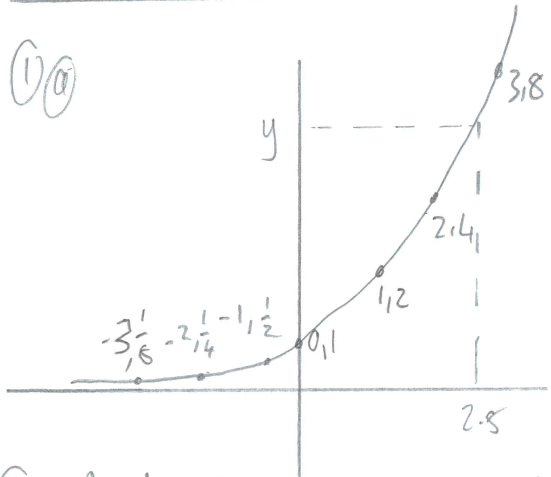
$$\therefore 32 - \frac{64}{3}$$

$$= \frac{32}{3}$$

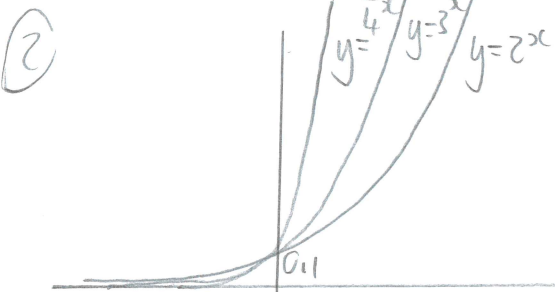
Year 1 ~~8C~~ Exponential Functions

← 4, 16

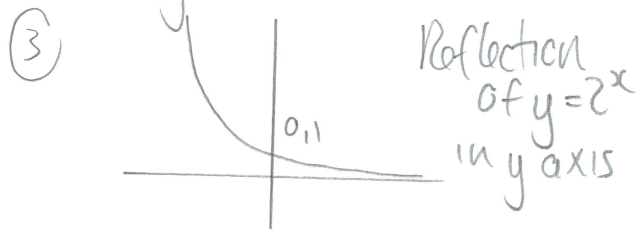
(i)(a)



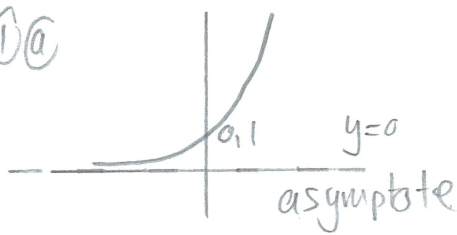
(b) Reads from graph as shown above giving ≈ 5.5 ish



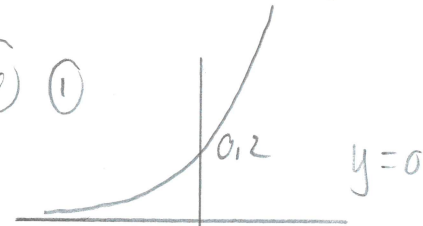
hard to see but $2^x > 3^x > 4^x$ for negative values



(i)(a)

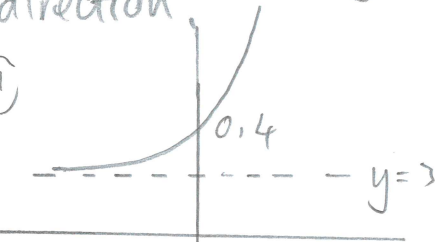


(b) (i)



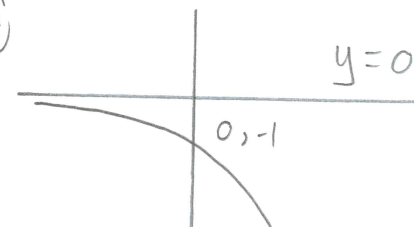
SF 2 stretch in +y direction

(ii)



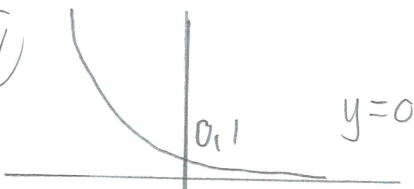
translation $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$

(iii)



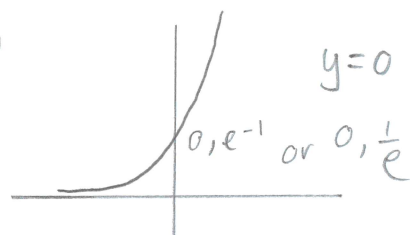
Reflection in x axis

(iv)



Reflection in y axis

(v)



Translation $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(2) Substitute coordinates into equation.

when $x=2, y=18$

$$\therefore 18 = pa^2$$

when $x=3, y=54$

$$\therefore 54 = pa^3$$

(b) $54 = pa^3$ (1)

$18 = pa^2$ (2)

(1) \div (2)

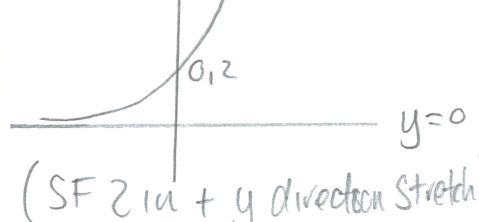
$$3 = a$$

Sub back in to (2)

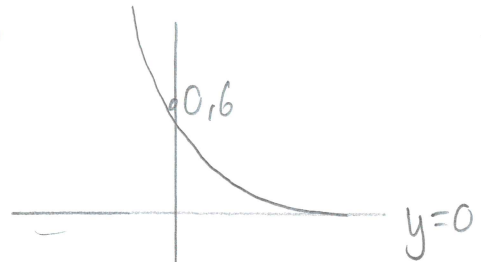
$$18 = p(3)^2$$

$$p = 2$$

(c) $y = 2 \times 3^x$



(i)



when $x=0, y = 3 \times (\frac{1}{2})^{-1}$
 $y = 6$

asymptote: $y=0$

(2) Sub in points

when $x=-1, y=0$

$$\therefore 0 = ab^{-1} + 2$$

or $\frac{a}{b} = -2$

or $a = -2b$ (1)

when $y = -2x = 0$

$$\therefore -2 = ab^0 + 2$$

$$-2 = a + 2$$

$$-4 = a$$
 (2)

If $a = -4, b = 2$

Equation: $y = -4 \times 2^x + 2$

At the point $(3, p)$

$$p = -4 \times 2^3 + 2$$

$$= -32 + 2$$

$$= -30 \checkmark$$

60 Continued.

$$(3) y = ab^x + c$$

Crosses y axis when
 $x=0$ so $y = ab^0 + c$

$$y = a(1) + c$$

$$y = a + c$$

$$\therefore \underline{\underline{(0, a+c)}}$$

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