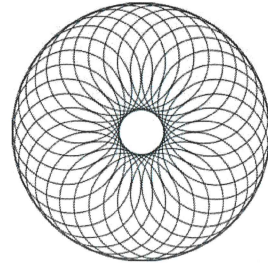


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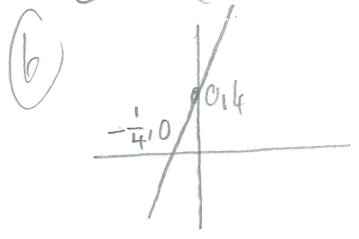
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<p>(1) Indices (2) Expanding Brackets (3) Factorising Expressions (4) More Indices (Negative and Fractional) (5) Working with Surds (6) Solving Quadratic Equations (7) Completing the Square for Quadratics Expressions (8) Function Notation (9) Sketching Quadratic Graphs (10) The Discriminant for Quadratic Equations (11) Applications of Quadratics Equations (12) Solving Linear Simultaneous Equations (13) Linear & Non-Linear Simultaneous Equations (14) Graphing Simultaneous Equations (15) Linear Inequalities (16) Quadratic Inequalities (17) Graphing Inequalities (18) Shading Inequalities (19) Cubic Graphs (20) Quartic Graphs (21) Reciprocal Graphs (22) The Intersection of Graphs (23) Transforming Graphs (Translations) (24) Transforming Graphs (Stretching/Reflecting) (25) Straight Line Graphs in the form $y = mx + c$ (26) More Straight Line Graphs (27) Straight Line Graphs (Parallel & Perpendicular) (28) The Geometry of Straight Lines (29) The Application of Linear Graphs (30) Circle Geometry Midpoint & Perpendicular</p>	<p>(31) The Equation of a Circle (32) Circles and Straight Lines (Intersections) (33) Circles (Tangents and Chords) (34) Circles and Triangles (35) Algebraic Fractions (36) Polynomial Division (37) The Factor and Remainder Theorem (38) An Introduction to Mathematical Proof (39) Methods of Proof (40) Binomial Expansion (Using Pascal's Triangle) (41) Binomial Expansion (Factorial Notation) (42) Binomial Expansion (The $\binom{n}{r}$ Method) (43) Binomial Expansion (Problem Solving) (44) Binomial Expansion (Estimations and Approximations) (45) The Cosine Rule (46) The Sine Rule (47) Areas of a Triangles (48) Triangles (Problem Solving) (49) Sine, Cosine & Tangent Graphs (50) Transforming Graphs (Trigonometry) (51) The 'CAST' Diagram for Trig Ratios (52) Trigonometry (Exact Values) (53) Proving Trigonometric Identities (54) Solving Basic Trigonometric Equations (55) More Challenging Trigonometric Equations (56) Using Identities to Solve Trig Equations (57) Vectors (Introduction)</p>	<p>(58) Vector Notation (Column and i and j form) (59) Vectors (Magnitude and Direction) (60) Vectors (Position and Direction Vectors) (61) Vector Geometry (62) Application of Vectors (63) Differentiation (Gradients of Curves) (64) Differentiation from 1st Principles (65) Differentiating x^n (Basic Powers of) (66) Differentiation (Quadratic Expression) (67) Differentiation (Multiple Terms) (68) Differentiation (Gradients, Tangents and Normals) (69) Differentiation (Increasing and Decreasing Functions) (70) Differentiation (Stationary Points) (71) Differentiation (Gradient Functions) (72) The Applications of Differentiation (73) Integration (Basic Expressions (x^n)) (74) Indefinite Integrals (75) Integration (Finding c and Finding Functions) (76) Integration (Definite Integrals) (77) Integration (Basic Areas Under Curves) (78) Integration ('Negative and Positive Areas') (79) Integration (Areas between Curves and Lines) (80) Basic Exponential Functions (81) 'The' Exponential Function $y = e^x$ (82) Applications of Basic Exponential Models (83) Logarithms (Simplifying & Evaluating) (84) Logarithms (The Log Laws) (85) Logarithms (Log and Exponential Equations)</p>
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71) Sketching Gradient Functions

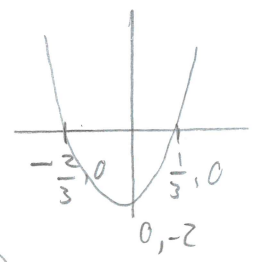
1) a) $f'(x) = 16x + 4$



2) a) $g'(x) = 9x^2 + 3x - 2$

b) $g'(x) = 0$ for S.P

$9x^2 + 3x - 2 = 0$
 $(3x+2)(3x-1) = 0$
 $x = -\frac{2}{3}, x = \frac{1}{3}$



3) Cubic.

1) a) $f'(x) = 24x^3 + 12x^2 - 24x - 12$

b) $0 = 24x^3 + 12x^2 - 24x - 12$

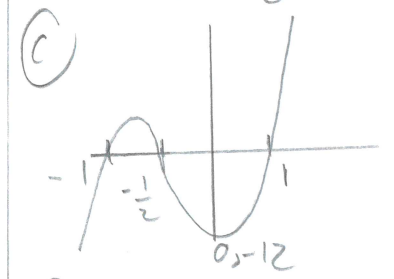
$0 = 2x^3 + x^2 - 2x - 1$

$f(1) = 0$ $\therefore x-1$ is a factor

$$\begin{array}{r} (x-1) \overline{2x^3 + x^2 - 2x - 1} \\ \underline{2x^3 - 2x^2} \\ 0 + 3x^2 - 2x - 1 \\ + 3x^2 - 3x \\ 0 + x - 1 \\ + x - 1 \\ 0 + 0 \end{array}$$

$(x-1)(2x^2 + 3x + 1) = 0$

$(x-1)(2x+1)(x+1) = 0$
 $x = 1, -\frac{1}{2}, -1$



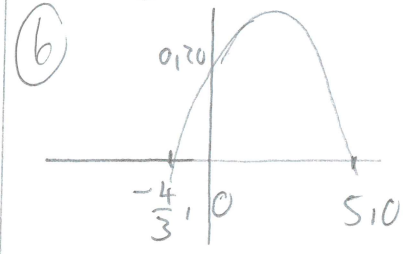
2) a) $g'(x) = -3x^2 + 11x + 20$

$g'(x) = 0$ for S.P

$0 = -3x^2 + 11x + 20$
 $3x^2 - 11x - 20 = 0$

$(3x+4)(x-5) = 0$

$x = -\frac{4}{3}, x = 5$ ✓



3) A quadratic function will have a linear gradient. Ax^2 is quadratic and the graph is a line.

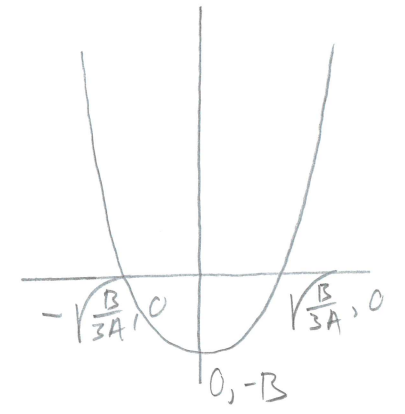
1) a) $A > 0$

b) The line crosses the x axis when the gradient = 0
 $f'(x) = 2Ax + B$

$0 = 2Ax + B$
 $x = -\frac{B}{2A}$

c) The constant doesn't change gradient.

2) $g'(x) = 3Ax^2 - B$



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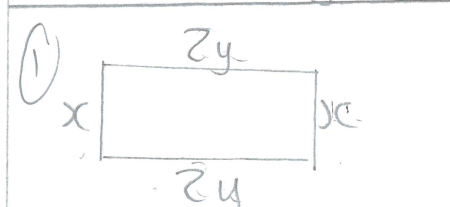
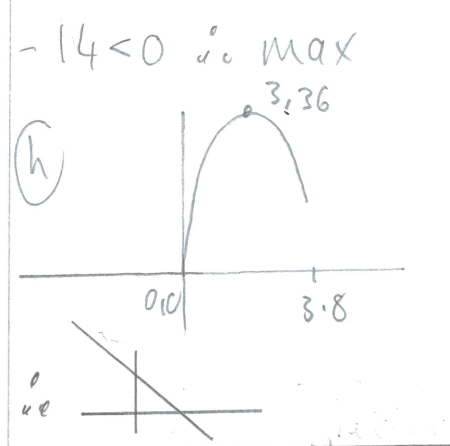
72 Modelling with Differentiation (Yr1)

1) a) $-t(t^2 - 2t - 15)$
 $-t(t-5)(t+3)$
 b) $h=0$ when at ground level:
 $0 = -t(t-5)(t+3)$
 $t=0, t=5, t=-3$
 as $0 \leq t \leq 3.8$
 $t \neq 5$.

c) $h'(t) = -3t^2 + 4t + 15$
 Stationary when $h'(t) = 0$
 $0 = 3t^2 - 4t - 15$
 $0 = (3t+5)(t-3)$
 $t \neq -\frac{5}{3} \therefore t = 3$

e) when $t=3$
 $n = -27 + 18 + 45$
 $n = 36$

f) $h''(t) = -6t + 4$
 g) $h''(3) = -18 + 4 = -14$
 $-14 < 0 \therefore$ max



a) $4y + 2x = 60$
 $2y + x = 30$
 $2y = 30 - x$

b) $A = lw$
 $A = 2xy$
 $A = x(30 - x)$

c) $A = 30x - x^2$
 $\frac{dA}{dx} = 30 - 2x$

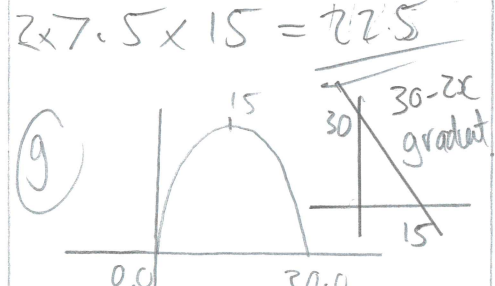
$\frac{dA}{dx} = 0$ for Max/Min

$0 = 30 - 2x$
 $x = 15$

d) $\frac{d^2A}{dx^2} = -2$

e) $-2 < 0 \therefore$ Max

f) when $x=15$
 $y = \frac{30-x}{2}$
 $y = \frac{15}{2}$
 $y = 7.5$
 $A = 15(30-15)$
 $A = 225$



h) Min = 0 when $x=0$. The rectangle wouldn't exist if the side was 0.

1) a) 0 m when $t=0$
 b) $x = -t^2 + t^{\frac{3}{2}} + 12t$
 $\frac{dx}{dt} = 0$ when the car is at a max distance
 $0 = -2t + \frac{3}{2}t^{\frac{1}{2}} + 12$
 $0 = 12 + \frac{3}{2}t^{\frac{1}{2}} - 2t$
 $0 = 24 + 3t^{\frac{1}{2}} - 4t$
 $A = 24, B = 3, C = -4$ o.e.

c) $0 = 4t - 3t^{\frac{1}{2}} - 24$
 $t^{\frac{1}{2}} = 2.853 \dots$ or $\frac{3 + \sqrt{393}}{8}$
 $t^{\frac{1}{2}} \neq -2.10 \dots$

$\therefore t = 8.139 \dots$
 when $t = 8.139 \dots$
 $x = 54.6$ m

d) $-t(t - t^{0.5} - 12) = 0$
 $t = 0$ or $t - t^{0.5} - 12 = 0$
 $(t^{\frac{1}{2}} - 4)(t^{\frac{1}{2}} + 3) = 0$
 $t \neq -3 \therefore t^{\frac{1}{2}} = 4 \Rightarrow t = 16$ BUT $0 \leq t \leq 12$

Y1 (73) Integrating

x^n Power

(a) $y = \int 4x dx$

$$y = \frac{4x^2}{2} + c$$

$$y = 2x^2 + c$$

(b) $y = \int 2x^2 dx$

$$y = \frac{2x^3}{3} + c$$

(c) $y = \int 4x^3 - 8x dx$

$$y = \frac{4x^4}{4} - \frac{8x^2}{2} + c$$

$$y = x^4 - 4x^2 + c$$

(d) $y = \int 5x^2 - x + 3 dx$

$$y = \frac{5x^3}{3} - \frac{x^2}{2} + 3x + c$$

(e) $y = \frac{5}{6}x^{3/2} + c$

$$y = \frac{5}{9}x^{3/2} + c$$

(2) (a) $f(x) = \int x^{3/2} dx$

$$f(x) = \frac{x^{5/2}}{\frac{5}{2}} + c$$

$$f(x) = \frac{2}{5}x^{5/2} + c$$

(b) $f(x) = \int 5x^{-2} dx$

$$= \frac{5x^{-1}}{-1} + c$$

$$= -5x^{-1} + c$$

$$\text{or } -\frac{5}{x} + c$$

(c) $f(x) = \int x^{1/2} dx$

$$f(x) = \frac{x^{3/2}}{\frac{3}{2}} + c$$

$$f(x) = \frac{2}{3}x^{3/2} + c$$

(3) (a) $\frac{dy}{dx} = (3x+2)(3x+2)$

$$= 9x^2 + 6x + 6x + 4$$

$$= 9x^2 + 12x + 4$$

(b) $y = \int 9x^2 + 12x + 4 dx$

$$y = \frac{9x^3}{3} + \frac{12x^2}{2} + \frac{4x}{1} + c$$

$$y = 3x^3 + 6x^2 + 4x + c$$

(1) (a) $\frac{dy}{dx} = 2x^{-2} + x^{1/3}$

$$y = \frac{2x^{-1}}{-1} + \frac{x^{4/3}}{\frac{4}{3}} + c$$

$$y = -2x^{-1} + \frac{3}{4}x^{4/3} + c$$

(b) $y = 8x^{3/4} - \frac{x^{7/2}}{7/2} + c$

$$y = \frac{32}{3}x^{3/4} - \frac{2}{7}x^{7/2} + c$$

(c) $\frac{dy}{dx} = x^{3/2}$

$$y = \frac{x^{5/2}}{\frac{5}{2}} + c$$

$$y = \frac{2}{5}x^{5/2} + c$$

(d) $\frac{dy}{dx} = 24x^{-3/5} + 3x^{2/5}$

$$y = \frac{24x^{1/5}}{\frac{1}{5}} + \frac{3x^{7/5}}{\frac{7}{5}} + c$$

$$y = 72x^{1/5} + \frac{15}{7}x^{7/5} + c$$

(2) (a) $f(x) = \frac{x^2}{x^{1/2}} - \frac{3x}{x^{1/2}} + \frac{8}{x^{1/2}}$

$$= x^{3/2} - 3x^{1/2} + 8x^{-1/2}$$

(b) $f(x) = \frac{x^{5/2}}{5/2} - \frac{3x^{3/2}}{3/2} + \frac{8x^{1/2}}{1/2} + c$

$$= \frac{2}{5}x^{5/2} - 2x^{3/2} + 16x^{1/2} + c$$

(3) $\frac{dy}{dx} = \frac{x^6 - 2x^3 + 1}{x^3}$

$$= \frac{x^6}{x^3} - \frac{2x^3}{x^3} + \frac{1}{x^3}$$

$$= x^3 - 2 + x^{-3}$$

$$\therefore y = \int x^3 - 2 + x^{-3} dx$$

$$= \frac{x^4}{4} - \frac{2x^1}{1} + \frac{x^{-2}}{-2} + c$$

$$= \frac{1}{4}x^4 - 2x - \frac{1}{2x^2} + c$$

7.3 (cont. med.)

$$① \frac{dy}{dx} = \frac{x + 2x^{\frac{1}{2}} + 4}{x^3}$$

$$= \frac{1}{x^2} + \frac{2}{x^{5/2}} + \frac{4}{x^3}$$

$$= x^{-2} + 2x^{-5/2} + 4x^{-3}$$

$$y = \int x^{-2} + 2x^{-5/2} + 4x^{-3} dx$$

$$y = \frac{-x^{-1}}{-1} + \frac{2x^{-3/2}}{-3/2} + \frac{4x^{-2}}{-2} + c$$

$$y = -x^{-1} - \frac{4}{3}x^{-3/2} - 2x^{-2} + c$$

o.e.

$$② g'(x) = 3x^{-3/2} - x'$$

$$g(x) = \int 3x^{-3/2} - x' dx$$

$$= \frac{3x^{-1/2}}{-1/2} - \frac{x^2}{2} + c$$

$$= -6x^{-1/2} - \frac{1}{2}x^2 + c$$

$$③ f'(x) = x^3 + 3x^2(x^{\frac{1}{3}}) + 3x(x^{\frac{2}{3}}) + x'$$

$$= x^3 + 3x^{\frac{7}{3}} + 3x^{\frac{5}{3}} + x'$$

$$\therefore f(x) = \frac{1}{4}x^4 + \frac{3x^{\frac{10}{3}}}{\frac{10}{3}} + \frac{3x^{\frac{8}{3}}}{\frac{8}{3}} + \frac{x^2}{2} + c$$

$$= \frac{1}{4}x^4 + \frac{9}{10}x^{\frac{10}{3}} + \frac{9}{8}x^{\frac{8}{3}} + \frac{1}{2}x^2 + c$$

74 Indefinite Integrals

$$\textcircled{a} \int x^{\frac{4}{5}} dx = \frac{x^{\frac{9}{5}}}{\frac{9}{5}} + c$$
$$= \frac{5}{9} x^{\frac{9}{5}} + c$$

$$\textcircled{b} \int x^2 + x - 2 dx$$
$$= \frac{x^3}{3} + \frac{x^2}{2} - 2x + c$$

$$\textcircled{c} \int 5x^{-3} dx = \frac{5x^{-2}}{-2} + c$$
$$= -\frac{5}{2} x^{-2} + c$$

$$\textcircled{d} \int 4x^{-7} = \frac{4x^{-6}}{-6} + c$$
$$= -\frac{2}{3} x^{-6} + c$$

② $p=6, q=9$
reverse differentiation!

$$\textcircled{3} \int \frac{7}{2} t^{-\frac{1}{3}} dt = \frac{7}{2} t^{\frac{2}{3}} + c$$
$$= \frac{21}{4} t^{\frac{2}{3}} + c$$

$$\textcircled{1} I = \frac{y^2}{2} - \frac{y^{-1}}{-1} + c$$

$$I = \frac{1}{2} y^2 + y^{-1} + c$$

$$I = \frac{1}{2} y^2 + \frac{1}{y} + c \checkmark$$

$$\textcircled{2} \int 2t - \frac{1}{2} t^{-\frac{3}{2}} dt$$
$$= \frac{2t^2}{2} - \frac{1}{2} t^{-\frac{1}{2}} + c$$
$$= t^2 + t^{-\frac{1}{2}} + c$$

$$\textcircled{3} \int Ax^2 + 2ABx + B^2$$

$$\therefore A^2 = 9 \quad 2AB = 12$$

$$A = \pm 3 \quad 6B = 12$$

$$A = 3 \checkmark \quad B = 2 \checkmark$$

$$\textcircled{1} \sum_{c=0}^8 \binom{8}{c} (-3x)^c = \sum_{c=0}^8 \binom{8}{c} (-3x)^c + \sum_{c=0}^8 \binom{8}{c} (-3x)^{c+1} + \sum_{c=0}^8 \binom{8}{c} (-3x)^{c+2}$$

$$1 \binom{8}{0} + 8 \binom{8}{1} (-3x) + 28 \binom{8}{2} (9x^2) + \dots$$

$$1 - 24x + 252x^2 + \dots$$

$$\textcircled{6} \int 1 - 24x + 252x^2 dx = \frac{xc^1}{1} - \frac{24xc^2}{2} + \frac{252x^3}{3} + c$$
$$= x - 12x^2 + 84x^3 + c$$

$$\textcircled{7} \int 4 - t^{-\frac{3}{2}} + t^{-2} dt = \frac{4t^1}{1} - \frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} + \frac{t^{-1}}{-1} + c$$
$$= 4t + 2t^{-\frac{1}{2}} - t^{-1} + c$$

75 Finding Functions

① $y = \int 3x^2 + 4x - 7 dx$
 $y = \frac{3x^3}{3} + \frac{4x^2}{2} - \frac{7x^1}{1} + c$
 $y = x^3 + 2x^2 - 7x + c$
 when $x=1, y=2$
 $\therefore 2 = (1) + 2(1) - 7(1) + c$
 $c = 6$
 $y = x^3 + 2x^2 - 7x + 6$ ✓

② (a) $\frac{4}{3}x^{3/2} + c$
 $= \frac{8}{9}x^{3/2} + c$

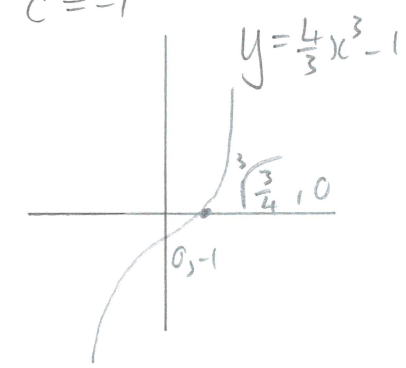
⑥ $f(x) = \frac{8}{9}x^{3/2} + c$
 $f(9) = 12$
 $\therefore 12 = \frac{8}{9}(9)^{3/2} + c$
 $12 = \frac{8}{9}(27) + c$
 $12 = 24 + c$
 $-12 = c$
 $\therefore f(x) = \frac{8}{9}x^{3/2} - 12$

③ $y = \int 2x^{-2} dx$
 $y = \frac{2}{-1}x^{-1} + c$
 $y = -\frac{2}{x} + c$
 when $x = -\frac{1}{4}, y = 8$
 $\therefore 8 = -\frac{2}{-\frac{1}{4}} + c$
 $c = 0$
 $y = -\frac{2}{x}$

① $\frac{dy}{dx} = 5x^{3/2}$
 $y = \int 5x^{3/2} dx$
 $y = \frac{5x^{5/2}}{\frac{5}{2}} + c$
 $y = 2x^{5/2} + c$
 when $x=1, y=3$
 $3 = 2(1) + c$
 $c = 1$
 $\therefore y = 2x^{5/2} + 1$

② $f(x) = \int 1 - 8x^{-3} dx$
 $= x - \frac{8x^{-2}}{-2} + c$
 $= x + 4x^{-2} + c$
 $= 4x^{-2} + x + c$
 when $x=1, y=0$
 $\therefore 0 = 4(1) + (1) + c$
 $c = -5$
 $\therefore f(x) = 4x^{-2} + x - 5$

③ (a) cubic
 (b) $y = \int 4x^2 dx$
 $y = \frac{4x^3}{3} + c$
 when $x=3, y=35$
 $\therefore 35 = \frac{4}{3}(27) + c$
 $c = -1$



① (a) $A = \int -3t^2 + 6t + 4 dt$
 $A = -t^3 + 3t^2 + 4t + c$
 when $t=0, A=0 \therefore c=0$
 $A = -t^3 + 3t^2 + 4t$
 (b) when $t=1, A = -1 + 3 + 4$
 $A = 6$
 (c) $\frac{dA}{dt} = 0$
 $0 = -3t^2 + 6t + 4$
 $t = 1 \pm \sqrt{\frac{7}{3}}$
 $t \neq 1 - \sqrt{\frac{7}{3}} \therefore t = 1 + \sqrt{\frac{7}{3}}$
 $= 2.53 \text{ seconds}$

② $\frac{dx}{dt} = 8t^{-2} - t^{-3}$
 (a) $\frac{dx}{dt}$
 $x = \int 8t^{-2} - t^{-3} dt$
 $x = \frac{8t^{-1}}{-1} - \frac{t^{-2}}{-2} + c$
 $x = -\frac{8}{t} + \frac{1}{2t^2} + c$
 $4 = -8 + \frac{1}{2} + c$
 $c = \frac{21}{2} \therefore x = -\frac{8}{t} + \frac{1}{2t^2} + \frac{21}{2}$

(75) Continued.

$$x = -\frac{8}{t} + \frac{1}{2t^2} + \frac{21}{2}$$

$$x = -\frac{8}{\frac{8}{2}} + \frac{1}{8} + \frac{21}{2}$$

$$x = \frac{53}{8}$$

(b) $f'(t) > 0$

$$\therefore 8t - 1 > 0$$

$$t > \frac{1}{8}$$

(c) $f'(t) < 0$

$$8t - 1 < 0$$

$$0 < t < \frac{1}{8}$$
