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A LEVEL MATHS

YEAR 1 PURE

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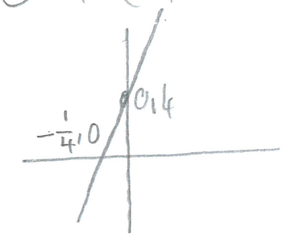
71 Sketching Gradient Functions

① ⓐ $f'(x) = 16x + 4$

ⓑ



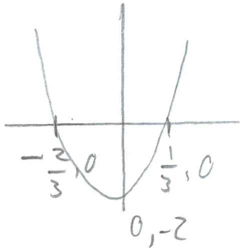
② ⓐ $g'(x) = 9x^2 + 3x - 2$

ⓑ $g'(x) = 0$ for S.P

$9x^2 + 3x - 2 = 0$

$(3x+2)(3x-1) = 0$

$x = -\frac{2}{3}$ $x = \frac{1}{3}$



③ Cubic.

① ⓐ $f'(x) = 24x^3 + 12x^2 - 24x - 12$

ⓑ $0 = 24x^3 + 12x^2 - 24x - 12$

$0 = 2x^3 + x^2 - 2x - 1$

$f(1) = 0 \therefore x - 1$ is a factor

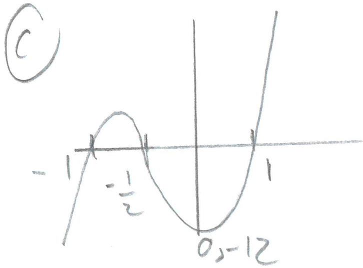
$$\begin{array}{r|l} & 2x^2 + 3x + 1 \\ \hline (x-1) & 2x^3 + x^2 - 2x - 1 \\ & 2x^3 - 2x^2 \\ \hline & 0 + 3x^2 - 2x \\ & +3x^2 - 3x \\ \hline & 0 + x - 1 \\ & +x - 1 \\ \hline & 0 + 0 \end{array}$$

$(x-1)(2x^2+3x+1) = 0$

$(x-1)(2x+1)(x+1) = 0$

$x = 1, -\frac{1}{2}, -1$

ⓒ



② ⓐ $g'(x) = -3x^2 + 11x + 20$

$g'(x) = 0$ for S.P

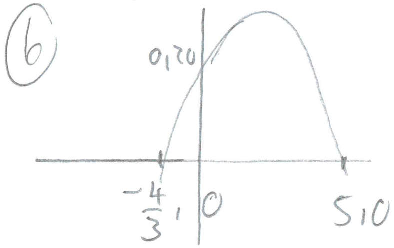
$0 = -3x^2 + 11x + 20$

$3x^2 - 11x - 20 = 0$

$(3x+4)(x-5) = 0$

$x = -\frac{4}{3}$ $x = 5$ ✓

ⓑ



③ A quadratic function will have a linear gradient Ax^2 is quadratic and the graph is a line.

① ⓐ $A > 0$

ⓑ The line crosses the x axis when the gradient $= 0$

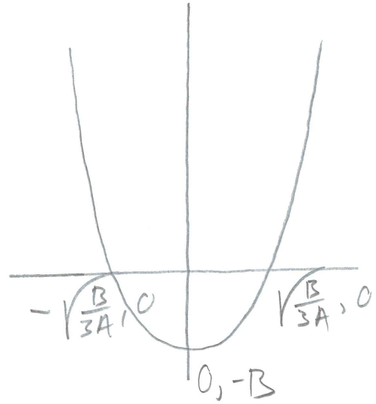
$f'(x) = 2Ax + B$

$\therefore 0 = 2Ax + B$

$x = -\frac{B}{2A}$

ⓒ The constant doesn't change gradient.

② $g'(x) = 3Ax^2 - B$



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(12) Modelling with Differentiation (Yr1)

1 (a) $-t(t^2-2t-15)$

$-t(t-5)(t+3)$

(b) $h=0$ when at ground level:

$0=-t(t-5)(t+3)$

$t=0$, $t=5$, $t\neq-3$

as $0\leq t\leq 3.8$

$t\neq 5$.

(c) $h'(t)=-3t^2+4t+15$

(d) Stationary when $h'(t)=0$

$0=3t^2-4t-15$

$0=(3t+5)(t-3)$

$t\neq-\frac{5}{3}$ $\therefore t=3$

(e) when $t=3$

$h=-27+18+45$

$h=36$

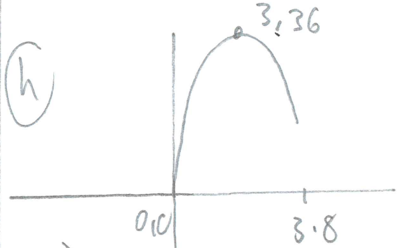
(f) $h''(t)=-6t+4$

(g) $h''(3)=-18+4$

$=-14$

$-14<0$ $\therefore$ max

(h)



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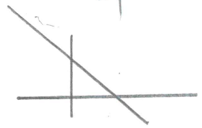

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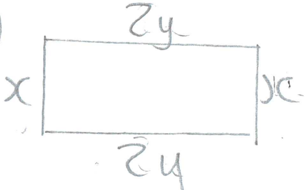


(a) $4y+2x=60$

$2y+x=30$

$2y=30-x$

(b) $A=lw$

$A=2xy$

$A=x(30-x)$

(c) $A=30x-x^2$

$\frac{dA}{dx}=30-2x$

$\frac{dA}{dx}=0$ for max/min

$0=30-2x$

$x=15$

(d) $\frac{d^2A}{dx^2}=-2$

(e) $-2<0$ $\therefore$ max

(f) when $x=15$

$y=\frac{30-x}{2}$

$y=\frac{15}{2}$

$y=7.5$

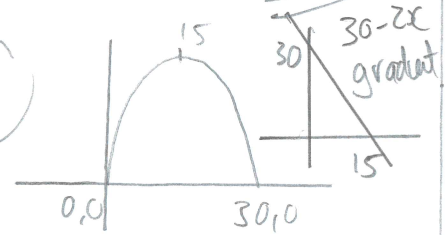
$A=15(30-15)$

$A=225$

or

$2\times7.5\times15=225$

(g)



(h) Min $=0$ when $x=0$. The rectangle wouldn't exist if the side was 0.

1 (a) 0m when $t=0$

(b) $x=-t^2+t^{\frac{3}{2}}+12t$

$\frac{dx}{dt}=0$ when the car is at a max distance

$0=-2t+\frac{3}{2}t^{\frac{1}{2}}+12$

$0=12+\frac{3}{2}t^{\frac{1}{2}}-2t$

$0=24+3t^{\frac{1}{2}}-4t$

$A=24$, $B=3$, $C=-4$ o.e.

(c) $0=4t-3t^{\frac{1}{2}}-24$

$t^{\frac{1}{2}}=2.853\ldots$ or $\frac{3+\sqrt{393}}{8}$

$t^{\frac{1}{2}}\neq-2.10\ldots$

$\therefore t=8.139\ldots$

when $t=8.139\ldots$

$x=54.6$m

(d) $-t(t-t^{0.5}-12)=0$

$t=0$ or $t-t^{0.5}-12=0$

$(t^{\frac{1}{2}}-4)(t^{\frac{1}{2}}+3)=0$

$t^{\frac{1}{2}}\neq-3$ $\therefore t^{\frac{1}{2}}=4 \Rightarrow t=16$ BUT $0\leq t\leq 12$

Y1 (13) Integrating

x^n Pure

1 (a) $y=\int 4x\,dx$

$y=\frac{4x^2}{2}+c$

$y=2x^2+c$

(b) $y=\int 2x^2\,dx$

$y=\frac{2x^3}{3}+c$

(c) $y=\int 4x^3-8x\,dx$

$y=\frac{4x^4}{4}-\frac{8x^2}{2}+c$

$y=x^4-4x^2+c$

(d) $y=\int 5x^2-x+3\,dx$

$y=\frac{5x^3}{3}-\frac{x^2}{2}+3x+c$

(e) $y=\frac{\frac{5}{6}x^{3/2}}{\frac{3}{2}}+c$

$y=\frac{5}{9}x^{3/2}+c$

2 (a) $f(x)=\int x^{3/2}\,dx$

$f(x)=\frac{x^{5/2}}{\frac{5}{2}}+c$

$f(x)=\frac{2}{5}x^{5/2}+c$

(b) $f(x)=\int 5x^{-2}\,dx$

$=\frac{5x^{-1}}{-1}+c$

$=-5x^{-1}+c$

or $-\frac{5}{x}+c$

(c) $f(x)=\int x^{\frac{1}{2}}\,dx$

$f(x)=\frac{x^{3/2}}{\frac{3}{2}}+c$

$f(x)=\frac{2}{3}x^{3/2}+c$

3 (a) $\frac{dy}{dx}=(3x+2)(3x+2)$

$=9x^2+6x+6x+4$

$=9x^2+12x+4$

(b) $y=\int 9x^2+12x+4\,dx$

$y=\frac{9x^3}{3}+\frac{12x^2}{2}+\frac{4x^1}{1}+c$

$y=3x^3+6x^2+4x+c$

---

1 (a) $\frac{dy}{dx}=2x^{-2}+x^{\frac{1}{3}}$

$y=\frac{2x^{-1}}{-1}+\frac{x^{\frac{4}{3}}}{\frac{4}{3}}+c$

$y=-2x^{-1}+\frac{3}{4}x^{\frac{4}{3}}+c$

(b) $y=\frac{8x^{\frac{3}{4}}}{\frac{3}{4}}-\frac{x^{\frac{7}{2}}}{\frac{7}{2}}+c$

$y=\frac{32}{3}x^{3/4}-\frac{2}{7}x^{\frac{7}{2}}+c$

(c) $\frac{dy}{dx}=x^{\frac{3}{2}}$

$y=\frac{x^{5/2}}{\frac{5}{2}}+c$

$y=\frac{2}{5}x^{5/2}+c$

(d) $\frac{dy}{dx}=24x^{-\frac{2}{3}}+3x^{\frac{2}{5}}$

$y=\frac{24x^{\frac{1}{3}}}{\frac{1}{3}}+\frac{3x^{7/5}}{\frac{7}{5}}+c$

$y=72x^{\frac{1}{3}}+\frac{15}{7}x^{7/5}+c$

2 (a) $f(x)=\frac{x^2}{x^{\frac{1}{2}}}-\frac{3x}{x^{\frac{1}{2}}}+\frac{8}{x^{\frac{1}{2}}}$

$=x^{3/2}-3x^{\frac{1}{2}}+8x^{-\frac{1}{2}}$

(b) $f(x)=\frac{x^{5/2}}{\frac{5}{2}}-\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{8x^{\frac{1}{2}}}{\frac{1}{2}}+c$

$=\frac{2}{5}x^{5/2}-2x^{3/2}+16x^{\frac{1}{2}}+c$

3 $\frac{dy}{dx}=\frac{x^6-2x^3+1}{x^3}$

$=\frac{x^6}{x^3}-\frac{2x^3}{x^3}+\frac{1}{x^3}$

$=x^3-2+x^{-3}$

$\therefore y=\int x^3-2+x^{-3}\,dx$

$=\frac{x^4}{4}-\frac{2x^1}{1}+\frac{x^{-2}}{-2}+c$

$=\frac{1}{4}x^4-2x-\frac{1}{2x^2}+c$

7·3 Continued.

① $\frac{dy}{dx} = \frac{x + 2x^{\frac{1}{2}} + 4}{x^3}$

$= \frac{1}{x^2} + \frac{2}{x^{5/2}} + \frac{4}{x^3}$

$= x^{-2} + 2x^{-5/2} + 4x^{-3}$

$y = \int x^{-2} + 2x^{-5/2} + 4x^{-3}\,dx$

$y = -\frac{x^{-1}}{1} + \frac{2x^{-\frac{3}{2}}}{-\frac{3}{2}} + \frac{4x^{-2}}{-2} + c$

$y = -x^{-1} - \frac{4}{3}x^{-\frac{3}{2}} - 2x^{-2} + c$

o.e.

② $g'(x) = 3x^{-\frac{3}{2}} - x^1$

$g(x) = \int 3x^{-3/2} - x^1\,dx$

$= \frac{3x^{-\frac{1}{2}}}{-\frac{1}{2}} - \frac{x^2}{2} + c$

$= -6x^{-\frac{1}{2}} - \frac{1}{2}x^2 + c$

③ $f'(x) = x^3 + 3x^2(x^{\frac{1}{3}}) + 3x(x^{\frac{2}{3}}) + x$

$= x^3 + 3x^{\frac{7}{3}} + 3x^{5/3} + x^1$

$\therefore f(x) = \frac{1}{4}x^4 + \frac{3x^{10/3}}{\frac{10}{3}} + \frac{3x^{8/3}}{\frac{8}{3}} + \frac{x^2}{2} + c$

$= \frac{1}{4}x^4 + \frac{9}{10}x^{10/3} + \frac{9}{8}x^{8/3} + \frac{1}{2}x^2 + c$

14 Indefinite Integrals

1 (a) $\int x^{\frac{4}{5}}dx = \frac{x^{9/5}}{\frac{9}{5}} + c$

$= \frac{5}{9}x^{9/5} + c$

(b) $\int x^2 + x - 2\,dx$

$= \frac{x^3}{3} + \frac{x^2}{2} - 2x + c$

(c) $\int 5x^{-3}dx = \frac{5x^{-2}}{-2} + c$

$= -\frac{5}{2}x^{-2} + c$

(d) $\int 4x^{-7} = \frac{4x^{-6}}{-6} + c$

$= -\frac{2}{3}x^{-6} + c$

2 $p = 6,\ q = 9$

reverse differentiation!

3 $\int \frac{7}{2}t^{-\frac{1}{3}}dt = \frac{\frac{7}{2}t^{\frac{2}{3}}}{\frac{2}{3}} + c$

$= \frac{21}{4}t^{\frac{2}{3}} + c$ ✓

1 $I = \frac{y^2}{2} - \frac{y^{-1}}{-1} + c$

$I = \frac{1}{2}y^2 + y^{-1} + c$

$I = \frac{1}{2}y^2 + \frac{1}{y} + c$ ✓

2 $\int 2t - \frac{1}{2}t^{-\frac{3}{2}}dt$

$= \frac{2t^2}{2} - \frac{\frac{1}{2}t^{-\frac{1}{2}}}{-\frac{1}{2}} + c$

$= t^2 + t^{-\frac{1}{2}} + c$

3 $\int A^2x^2 + 2ABx + B^2$

$\therefore A^2 = 9 \qquad 2AB = 12$

$A = \pm 3 \qquad 6B = 12$

$A = 3$ ✓ $\qquad B = 2$ ✓

1 $\binom{8}{0}(1)^8(-3x)^0 + \binom{8}{1}(1)^7(-3x)^1 + \binom{8}{2}(1)^6(-3x)^2$

$1(1)(1) + 8(1)(-3x) + 28(1)(9x^2)$

$1 \quad -24x + 252x^2 + \ldots$

(b) $\int 1 - 24x + 252x^2dx = \frac{x^1}{1} - \frac{24x^2}{2} + \frac{252x^3}{3} + c$

$= x - 12x^2 + 84x^3 + c$

2 $\int 4 - t^{-\frac{3}{2}} + t^{-2}dt = \frac{4t^1}{1} - \frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} + \frac{t^{-1}}{-1} + c$

$= 4t + 2t^{-\frac{1}{2}} - t^{-1} + c$

(75) Finding Functions

① $y=\int 3x^2+4x-7\,dx$

$y=\frac{3x^3}{3}+\frac{4x^2}{2}-\frac{7x^1}{1}+c$

$y=x^3+2x^2-7x+c$

when $x=1$, $y=2$

$\therefore\ 2=(1)+2(1)-7(1)+c$

$c=6$

$y=x^3+2x^2-7x+6$ ✓

② ⓐ $\dfrac{\frac{4}{3}x^{3/2}}{\frac{3}{2}}+c$

$=\frac{8}{9}x^{3/2}+c$

ⓑ $f(x)=\frac{8}{9}x^{3/2}+c$

$f(9)=12$

$\therefore\ 12=\frac{8}{9}(9)^{3/2}+c$

$12=\frac{8}{9}(27)+c$

$12=24+c$

$-12=c$

$\therefore\ f(x)=\frac{8}{9}x^{3/2}-12$

③ $y=\int 2x^{-2}\,dx$

$y=\frac{2}{-1}x^{-1}+c$

$y=-\frac{2}{x}+c$

when $x=-\frac{1}{4}$ $y=8$

$\therefore\ 8=\dfrac{-2}{-\frac{1}{4}}+c$

$c=0$

$y=-\frac{2}{x}$

① $\frac{dy}{dx}=5x^{3/2}$

$y=\int 5x^{3/2}\,dx$

$y=\dfrac{5x^{5/2}}{\frac{5}{2}}+c$

$y=2x^{5/2}+c$

when $x=1$ $y=3$

$3=2(1)+c$

$c=1$

$\therefore\ y=2x^{5/2}+1$

② $f(x)=\int 1-8x^{-3}\,dx$

$=x-\frac{8x^{-2}}{-2}+c$

$=x+4x^{-2}+c$

$=4x^{-2}+x+c$

when $x=1$ $y=0$

$\therefore\ 0=4(1)+(1)+c$

$c=-5$

$\therefore\ f(x)=4x^{-2}+x-5$

③ ⓐ Cubic

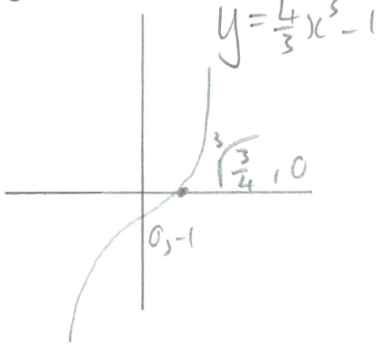
ⓑ $y=\int 4x^2\,dx$

$y=\frac{4x^3}{3}+c$

when $x=3$, $y=35$

$\therefore\ 35=\frac{4}{3}(27)+c$

$c=-1$



① ⓐ $A=\int -3t^2+6t+4\,dt$

$A=-t^3+3t^2+4t+c$

when $t=0$ $A=0$ $\therefore\ c=0$

$A=-t^3+3t^2+4t$

ⓑ when $t=1$ $A=-1+3+4$

$A=6$

ⓒ $\frac{dA}{dt}=0$

$0=-3t^2+6t+4$

$t=1\pm\sqrt{\frac{7}{3}}$

$t\neq 1-\sqrt{\frac{7}{3}}$ $\therefore\ t=1+\sqrt{\frac{7}{3}}$

$=2.53$ seconds

② $\frac{dx}{dt}=8t^{-2}-t^{-3}$

ⓐ $x=\int 8t^{-2}-t^{-3}\,dt$

$x=\frac{8t^{-1}}{-1}-\frac{t^{-2}}{-2}+c$

$x=-\frac{8}{t}+\frac{1}{2t^2}+c$

$4=-8+\frac{1}{2}+c$

$c=\frac{21}{2}$ $\therefore\ x=-\frac{8}{t}+\frac{1}{2t^2}+\frac{21}{2}$

15 Continued

$$x = -\frac{8}{t} + \frac{1}{2t^2} + \frac{21}{2}$$

$$x = -\frac{8}{2} + \frac{1}{8} + \frac{21}{2}$$

$$x = \frac{53}{8}$$

(b) $f'(t) > 0$

$\therefore 8t - 1 > 0$

$t > \frac{1}{8}$

(c) $f'(t) < 0$

$8t - 1 < 0$

$0 < t < \frac{1}{8}$