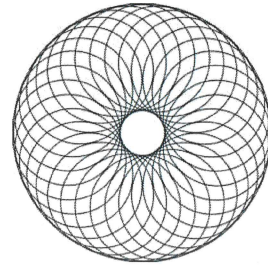


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# 66 Differentiating Quadratics (Purely 1)

1) a)  $\frac{dy}{dx} = 2x + 3$

b)  $\frac{dy}{dx} = 2x - 2$

c)  $\frac{dy}{dx} = -2x + 6$

d)  $\frac{dy}{dx} = 8x - 3$

e)  $f'(x) = 10x - 1$

f)  $f'(x) = -6x$

3) a)  $f'(x) = 8x + 2$

$f'(2) = 8(2) + 2 = 18$

b)  $8x + 2 = 34$   
 $8x = 32$   
 $x = 4$

1) a)  $y = x^2 - 4x$   
 $\therefore \frac{dy}{dx} = 2x - 4$

b)  $y = x^2 + x - 12$

$\frac{dy}{dx} = 2x + 1$

c)  $y = 6x^2 + 7x - 5$

$\therefore \frac{dy}{dx} = 12x + 7$

d)  $y = x^2 - 6x + 9$

$\therefore \frac{dy}{dx} = 2x - 6$

2) a)  $g(x) = 6x^2 - 24x$

$\therefore g'(x) = 12x - 24$

b)  $g(x) = 16x^2 - 24x + 9$

$g'(x) = 32x - 24$

3)  $y = 16 - 40x + 25x^2$

$\frac{dy}{dx} = -40 + 50x$

$\therefore -40 + 50x = 5$   
 $x = \frac{9}{10}$

1) a)  $f(2) = 18$

$\therefore 18 = 4 + 2p + q$   
 $14 = 2p + q$  ①

$f(-3) = -27$

$\therefore -27 = 9 - 3p + q$   
 $-36 = -3p + q$  ②

① - ②

$50 = 5p$   
 $p = 10$

$\therefore q = -6$

b)  $f(x) = x^2 + 10x - 6$

$f'(x) = 2x + 10$

$-8 = 2a + 10$

$a = -9$

When  $x = -9$

$y = -15$

$\therefore b = -15$

c)  $\frac{dy}{dx} = 0$   $(-5, -31)$

$\therefore 2x + 10 = 0$   
 $x = -5$   $y = -31$

2)  $\frac{dx}{dt} = 8t - 8$

$\therefore -40 = 8p - 8$

$-32 = 8p$

$p = -4$

$x = 4t^2 - 8t$

When  $t = -4$

$x = 4(-4)^2 - 8(-4)$   
 $= 96$

$\therefore q = 96$

3)  $y = -[4 - 12x + 9x^2]$

$y = -4 + 12x - 9x^2$

$\frac{dy}{dx} = 12 - 18x$

$6 = 12 - 18x$

$18x = 6$

$x = \frac{1}{3}$

$y = -(2 - 3(\frac{1}{3}))^2$

$y = -(1)^2$

$y = -1$

$\therefore (\frac{1}{3}, -1)$

# (67) Differentiating with 2 or more terms

①  $\frac{dy}{dx} = 7x^6 - 3$

②  $y = x^8 - x^7$

$\therefore \frac{dy}{dx} = 8x^7 - 7x^6$

③  $y = 4x + x^{\frac{1}{2}} + 1$

$\frac{dy}{dx} = 4 + \frac{1}{2}x^{-\frac{1}{2}}$

or  $4 + \frac{1}{2\sqrt{x}}$

④  $y = x^{5/2} - 3x^{3/2}$

$\frac{dy}{dx} = \frac{5}{2}x^{3/2} - \frac{9}{2}x^{1/2}$

⑤  $f(x) = 7x^{2/5} - 4x^{-1}$

$f'(x) = \frac{14}{5}x^{-3/5} + 4x^{-2}$

or  $= \frac{14}{5x^{3/5}} + \frac{4}{x^2}$

⑥  $f(x) = 2x^{17/11} - 3x^{6/11}$

$f'(x) = \frac{34}{11}x^{6/11} - \frac{18}{11}x^{-5/11}$

①  $f(x) = 24 + 8x^{-1}$

$f'(x) = -8x^{-2}$

or  $-\frac{8}{x^2}$

②  $f'(x) = \frac{3}{5}x^{-6/5} + \frac{8}{3}x^{-2/3}$

③  $x = (t^3 t^{\frac{1}{2}}) + 10t^{-2}$

$x = t^{3\frac{1}{2}} + 10t^{-2}$

$\frac{dx}{dt} = \frac{3}{2}t^{\frac{1}{2}} - 20t^{-3}$

$= \frac{3}{2}\sqrt{t} - \frac{20}{t^3}$  ✓

④  $y = 8x + x^{-1}$

$\frac{dy}{dx} = 8 - x^{-2}$

$0 = 8 - x^{-2}$

$0 = 8 - \frac{1}{x^2}$

$\frac{1}{x^2} = 8$

$\frac{1}{8} = x^2$

$x = \pm \frac{1}{2\sqrt{2}}$

⑤  $f'(x) = 6x^{-\frac{1}{4}} - x^{-\frac{1}{2}}$

or  $\frac{6}{x^{\frac{1}{4}}} - \frac{1}{\sqrt{x}}$

when  $x=16$   
 $f'(16) = \frac{6}{16^{\frac{1}{4}}} - \frac{1}{\sqrt{16}}$   
 $= \frac{6}{2} - \frac{1}{4}$   
 $= \frac{12}{4} - \frac{1}{4}$   
 $= \frac{11}{4}$

③  $\frac{dy}{dx} = x^2 + x - 6$   
 $= (x+3)(x-2)$

⑥  $x = -3$  or  $x = 2$

①  $\frac{dy}{dx} = -x^{\frac{1}{2}} + x + 42$   
 $0 = -x^{\frac{1}{2}} + x + 42$   
 $x^{\frac{1}{2}} - x - 42 = 0$

$(x^{\frac{1}{2}} - 7)(x^{\frac{1}{2}} + 6) = 0$

$x^{\frac{1}{2}} = 7$   $x^{\frac{1}{2}} \neq -6$

$\therefore x = 49$   $y = \frac{6517}{6}$

②  $f'(x) = 0 - \frac{1}{2}x^{-\frac{1}{2}}$

$f'(x) = 0$  for stationary point.

$0 \neq -\frac{1}{2\sqrt{x}} \therefore$  no real solutions

③  $\frac{dy}{dx} = 2ax + b$

①  $\text{SP } 0 = 2a(-\frac{3}{8}) - \frac{3}{8}(b)$

$\therefore b = -2a$  ①

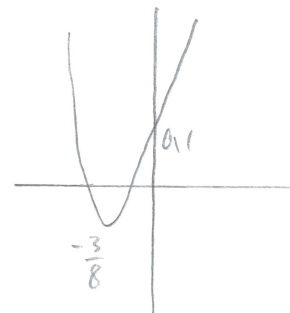
• y axis  $1 = c$  ②

• Gradient  $-5 = 2a(-1) + b$

$b = 2a - 5$  ③

① = ③  $-2a = 2a - 5$

$a = \frac{5}{4}$   $b = -\frac{5}{2}$   $c = 1$



# (68) Gradients, Tangents and Normals (Yin Pure)

①  $y = 4(1) + 2(1) + 1$

②  $y = 7$

③  $\frac{dy}{dx} = 12x^2 + 2$

④ when  $x=1$ ,  $\frac{dy}{dx} = 12(1) + 2 = 14$

⑤  $y - y_1 = m(x - x_1)$

$y - 7 = 14(x - 1)$

$y - 7 = 14x - 14$

$y = 14x - 7$  ✓

⑥  $-\frac{1}{14}$

⑦  $y - 7 = -\frac{1}{14}(x - 1)$

$14y - 98 = -x + 1$

$x + 14y = 99$  ✓

⑧  $\frac{dy}{dx} = 12x^2 - 10x$

⑨ when  $x=2$ ,  $\frac{dy}{dx} = 12(4) - 10(2) = 28$

⑩ when  $x=2$ ,  $y = 4(8) - 5(4) + 2 = 24$

$m = 28$

Point  $(2, 24)$

$y - 24 = 28(x - 2)$

$y = 28x - 56 + 24$

$y = 28x - 32$

⑪  $m = -\frac{1}{28}$

⑫ when  $x=3$ ,  $y = 4(27) - 5(9) + 2 = 65$

$\therefore (3, 65)$

$y - 65 = -\frac{1}{28}(x - 3)$  o.e.

⑬  $m = 3$

$\frac{dy}{dx} = 2x + 6$

$3 = 2x + 6$

$-3 = 2x$

$-\frac{3}{2} = x$

$y = \left(-\frac{3}{2}\right)^2 + 6\left(-\frac{3}{2}\right) = \frac{9}{4} - 9 = -\frac{27}{4}$

$\therefore y + \frac{27}{4} = 3\left(x - \frac{3}{2}\right)$

$y = 3x - \frac{9}{2} - \frac{27}{4}$

$y = 3x - \frac{45}{4}$

⑭  $y = x^{-1}$

$\frac{dy}{dx} = -x^{-2}$

or  $-\frac{1}{x^2}$

when  $x=2$

$y = \frac{1}{2}$  and  $\frac{dy}{dx} = -\frac{1}{4}$

$\therefore y - \frac{1}{2} = -\frac{1}{4}(x - 2)$

$y = -\frac{1}{4}x + 1$

⑮ when  $x=4$

$\frac{dy}{dx} = -\frac{1}{16}$

$\therefore$  normal  $m = 16$

$y - \frac{1}{4} = 16(x - 4)$

$y = 16x - 64 + \frac{1}{4}$

$y = 16x - \frac{255}{4}$

⑯  $\frac{dy}{dx} = 10x^4 + 1$

$11 = 10x^4 + 1$

$10x^4 = 10$

$x^4 = 1$

$x = \pm 1$

$x = 1$  as  $p > 0$

$\therefore$  when  $x=1$ ,  $y = 2(1) + 1 = 3$

⑰ when  $x=-3$

$y = 9 \therefore (-3, 9)$

$\frac{dy}{dx} = 2x$

when  $x=-3$

$\frac{dy}{dx} = -6$  Normal  $m = \frac{1}{6}$

$\therefore y - 9 = \frac{1}{6}(x + 3)$

when  $x=0$

$y - 9 = \frac{1}{6}$

$y = \frac{19}{6}$

when  $y=0$

$-54 = x + 3$

$x = -57$



$\sqrt{(-57-0)^2 + (0-\frac{19}{6})^2}$

$= \frac{19\sqrt{37}}{6}$  ✓

⑱  $y = 2x^{3/2}$

$\frac{dy}{dx} = 3x^{1/2}$

$m$  of line  $= -\frac{1}{18}$

$\therefore$  normal  $m = 18$

$18 = 3x^{1/2}$

$6 = x^{1/2}$

$36 = x$

when  $x=36$ ,  $y = 72(6) = 432$

$y - 432 = 18(x - 36)$

$0: -432 = 18(x - 36)$

$-24 = x - 36$

$x = 12$

$\therefore 12, 0$

⑲  $y = -x^2 + 3x$

$\frac{dy}{dx} = -2x + 3$

when  $x=2$ ,  $y = 2$

and  $\frac{dy}{dx} = -1$

$\therefore$   $m$  of normal  $= 1$

$y - 2 = 1(x - 2)$

$y = x$

Simultaneous equations

$x = -x(x - 3)$

$x = -x^2 + 3x$

$x^2 - 2x = 0$

$x(x - 2) = 0$

$x = 0$  or  $x = 2$   
 $y = 0$  or  $y = 2$

68) Continued

$$39) \frac{dy}{dx} = x^{-\frac{1}{2}} + 2$$

$$\frac{dy}{dx} = 6$$

$$\therefore 6 = \frac{1}{\sqrt{x}} + 2$$

$$4 = \frac{1}{\sqrt{x}}$$

$$\sqrt{x} = \frac{1}{4}$$

$$x = \frac{1}{16}$$

$$y = 2\sqrt{\frac{1}{16}} + 2\left(\frac{1}{16}\right) - 8$$
$$= \frac{1}{2} + \frac{1}{8} - 8$$
$$= -\frac{59}{8}$$

$$\therefore \left(\frac{1}{16}, -\frac{59}{8}\right)$$

$$6) y + \frac{59}{8} = 6\left(x - \frac{1}{16}\right)$$

$$y = 0 \quad x = \frac{119}{16}$$

$$x = 0 \quad y = \frac{31}{4}$$

$$\therefore \frac{1}{2} \left(\frac{119}{16}\right) \left(\frac{31}{4}\right) = \frac{3689}{128}$$

# (69) Increasing and Decreasing Functions

①  $f'(x) = 6x - 12$

$f(x)$  Stationary when  $f'(x) = 0$

$6x - 12 = 0$

$6x = 12$   
 $x = 2$

Either T.P

Min point  
 $2, -11$

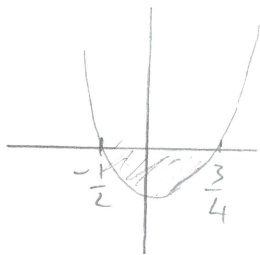
or  $6x - 12 > 0$   
 $6x > 12$   
 $x > 2$

② a)  $f'(x) = 8x^2 - 2x - 3$

$f'(x) < 0$  for decreasing

$8x^2 - 2x - 3 < 0$

$(4x-3)(2x+1) < 0$



$-\frac{1}{2} < x < \frac{3}{4}$

① a)  $f'(x) > 0$  for increasing

$f'(x) = 3ax^2 - 1$

$3ax^2 - 1 > 0$

$3(a)(4) - 1 > 0$

$12a - 1 > 0$

$a > \frac{1}{12}$

② The constant  $b$  differentiates to 0.

② a)  $f'(x) = 2x^2 - 2x - 12$

$= 2(x^2 - x - 6)$

$= 2(x-3)(x+2)$

② b) Station  $f'(x) = 0 \therefore$

$x = 3, x = -2$

①

decreasing  $f'(x) < 0$



$\therefore -2 < x < 3$

② increases  $f'(x) > 0$

$\therefore x < -2$  or  $x > 3$

③  $f'(x) > 0$  for increasing

$f(x) = x + xc^{-1}$

$f'(x) = 1 - x^{-2}$

$f'(x) = 1 - \frac{1}{x^2}$

$1 - \frac{1}{x^2} > 0$

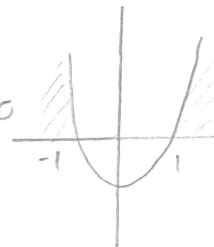
$1 > \frac{1}{x^2}$

$x^2 > 1$

$x^2 - 1 > 0$

$(x+1)(x-1) > 0$

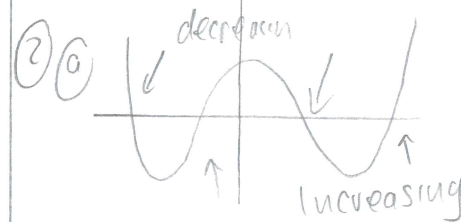
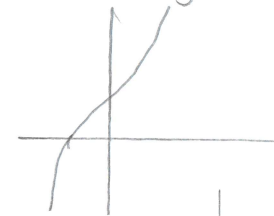
$x < -1$   
or  $x > 1$



$= -92$

$\therefore$  No roots for  $f'(x)$  and not stationary.

As it's a positive cubic it's always increasing.



a) 2

b) 2

① a)  $f(-\frac{1}{2}) = 2(-\frac{1}{8}) + 5(\frac{1}{4}) + 8(-\frac{1}{2})$

+3

$= -\frac{1}{4} + \frac{5}{4} - 4 + 3$

$= 1 - 4 + 3$

$= 0 \checkmark$

② Polynomial Division

$(2x+1)(x^2+2x+3)$

①  $f'(x) = 6x^2 + 10x + 8$

a)  $b^2 - 4ac = 100 - 4(6)(8)$

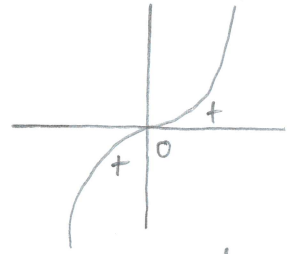
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Stationary Points

- ①  $f'(x) = 6x^2 + 8x$
- ②  $f''(x) = 0$  for S.P  
 $\therefore 0 = 6x^2 + 8x$   
 $0 = 2x(3x + 4)$   
 $x = 0$  or  $x = -\frac{4}{3}$  ✓
- ③ when  $x = 0$   $f(x) = 0$   
 $\therefore (0, 0)$   
 when  $x = -\frac{4}{3}$   $f(x) = \frac{64}{27}$   
 $\therefore (-\frac{4}{3}, \frac{64}{27})$
- ④  $f''(x) = 12x + 8$
- ⑤  $f''(0) = 8 \therefore$  min  
 $f''(-\frac{4}{3}) = -8 \therefore$  max
- ⑥  $\rightarrow$
- ⑦  $\frac{dy}{dx} = 20x^4$

- ⑧  $\frac{dy}{dx} = 0$  for SP.  
 $0 = 20x^4$   
 $x = 0$   
 when  $x = 0$   $y = 0$   
 $(0, 0)$ .
- ⑨ when  $x = -0.01$   
 $\frac{dy}{dx} = 2 \times 10^{-7}$  (+)
- when  $x = 0.01$   
 $\frac{dy}{dx} = 2 \times 10^{-7}$  (+)

$\therefore$  Point of inflexion



The gradient is the same either side of the stationary point  
 $\therefore$  point of inflexion (Point is close enough to SP)

- ⑩  $\frac{dy}{dx} = 2\sqrt{x} - 18$   
 for SP  $\frac{dy}{dx} = 0$   
 $\therefore 0 = 2\sqrt{x} - 18$   
 $0 = \sqrt{x} - 9$   
 $9 = \sqrt{x}$   
 $81 = x$   
 when  $x = 81$   
 $y = \frac{4}{3}(81)^{3/2} - 18(81)$   
 $= -486$   
 $\therefore (81, -486)$  is SP

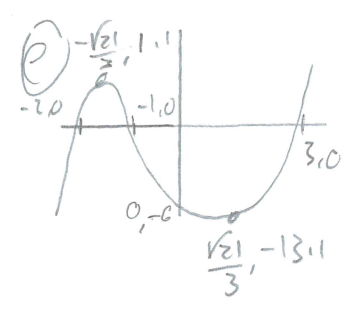
- ⑪  $\frac{d^2y}{dx^2} = x^{-1/2}$   
 or  $= \frac{1}{\sqrt{x}}$   
 when  $x = 81$   
 $\frac{d^2y}{dx^2} = \frac{1}{9}$   
 $\frac{1}{9} > 0 \therefore$  min.

⑫  $(x+1)(x^2 - x - 6)$   
 $x^3 - x^2 - 6x + x^2 - x - 6$

- $\therefore f(x) = x^3 - 7x - 6$
- ⑬  $\frac{dy}{dx} = 3x^2 - 7$   
 For S.P  $\frac{dy}{dx} = 0$   
 $\therefore 0 = 3x^2 - 7$   
 $7 = 3x^2$   
 $\frac{7}{3} = x^2$   
 $\pm \sqrt{\frac{7}{3}} = x$   
 $\pm \frac{\sqrt{21}}{3} = x$

- ⑭ when  $x = \frac{\sqrt{21}}{3}$   
 $y = -13.1$   
 when  $x = -\frac{\sqrt{21}}{3}$   
 $y = 1.13$

- ⑮  $f''(x) = 6x$   
 when  $x = \frac{\sqrt{21}}{3}$   
 $f''(x) = 2\sqrt{21}$   
 min as  $\rightarrow 0$   
 when  $x = -\frac{\sqrt{21}}{3}$   
 $f''(x) = -2\sqrt{21}$   
 max as  $0 <$



⑯  $f(x) = 0$   
 $\therefore x = 2$  is a root.  
 If  $x^2 + 5x + 10$  has roots  $b^2 - 4ac \geq 0$   
 $(5)^2 - 4(1)(10) = -15$   
 $\therefore$  no real roots.  
 $x = 2$  is the only real root.

- ⑰  $y = x^3 + 3x^2 - 20$   
 $\frac{dy}{dx} = 3x^2 + 6x$   
 for SP  $\frac{dy}{dx} = 0$   
 $\therefore 0 = 3x^2 + 6x$   
 $0 = 3x(x + 2)$   
 $x = 0$  or  $x = -2$   
 when  $x = 0$ ,  $y = -20$   
 $(0, -20)$   
 when  $x = -2$   $y = -16$   
 $(-2, -16)$

10. (10) Contin

(2) (c)  $\frac{d^2y}{dx^2} = 6x + 6$

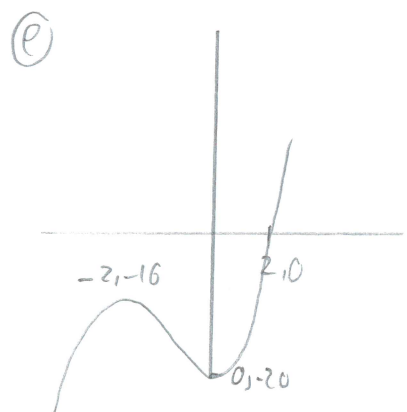
(a) when  $x = -2$

$\frac{dy}{dx} = -12 + 6$   
 $\frac{d^2y}{dx^2} = -6$

∴ max as  $0 <$   
when  $x = 0$

$\frac{d^2y}{dx^2} = 6$

∴ min as  $> 0$



(1) (a)  $y = \frac{x^{\frac{4}{3}} - x^{\frac{1}{2}}}{x^{\frac{1}{2}}}$

$y = x^{5/6} - x^{-1/2}$

$\frac{dy}{dx} = \frac{5}{6}x^{-1/6} - \frac{1}{2}x^{-3/2}$

$= \frac{1}{6}x^{-1/6} \left( 5 - 3x^{-1/3} \right)$

(b) for SP  $\frac{dy}{dx} = 0$

as  $x > 0$   $\frac{1}{6}x^{-1/6} \neq 0$

∴  $5 - 3x^{-1/3} = 0$

$3x^{-1/3} = 5$

$x^{-1/3} = \frac{5}{3}$

$x = \frac{27}{125}$

$x = \frac{27}{125}$

(c)  $\frac{d^2y}{dx^2} = -\frac{5}{36}x^{-7/6} + \frac{1}{4}x^{-3/2}$

when  $x = \frac{27}{125}$

$\frac{d^2y}{dx^2} \approx 1.66$  ∴ min as  $> 0$

(2)  $g'(x) = 8x^3 + 64$

$g'(x) = 0$  for min

$0 = 8x^3 + 64$

$x^3 = -8$

$x = -2$

when  $x = -2$

$g(x) = -96$

∴ least value = -96

(3) [ $f'(x) > 0$  for all values.]

$f'(x) = 3x^2 - 6x + 18$

If there is a turning point  $f'(x) = 0$

$0 = 3x^2 - 6x + 18$

$0 = x^2 - 2x + 6$

$b^2 - 4ac = (-2)^2 - 4(1)(6)$   
 $= -20$

∴ no turning points  
+ cubic ∴ gradient is always +.