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A LEVEL MATHS

YEAR 1 PURE

ANSWERS

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66 Differentiating Quadratics (Pure Y1)

1 a $\frac{dy}{dx} = 2x+3$

b $\frac{dy}{dx} = 2x-2$

c $\frac{dy}{dx} = -2x+6$

d $\frac{dy}{dx} = 8x-3$

2 a $f'(x) = 10x-1$

b $f'(x) = -6x$

3 a $f'(x) = 8x+2$

$f'(2) = 8(2)+2$
$= 18$

b $8x+2 = 34$
$8x = 32$
$x = 4$

1 a $y = x^2-4x$
$\therefore \frac{dy}{dx} = 2x-4$

b $y = x^2+x-12$
$\frac{dy}{dx} = 2x+1$

c $y = 6x^2+7x-5$
$\therefore \frac{dy}{dx} = 12x+7$

d $y = x^2-6x+9$
$\therefore \frac{dy}{dx} = 2x-6$

2 a $g(x) = 6x^2\ 24x$
$\therefore g'(x) = 12x-24$

b $g(x) = 16x^2-24x+9$
$g'(x) = 32x-24$

3 $y = 16-40x+25x^2$
$\frac{dy}{dx} = -40+50x$
$\therefore -40+50x = 5$
$x = \frac{9}{10}$

1 a $f(2) = 18$
$\therefore 18 = 4+2p+q$
$14 = 2p+q$ (1)

$f(-3) = -27$
$\therefore -27 = 9-3p+q$
$-36 = -3p+q$ (2)

(1) − (2)
$50 = 5p$
$p = 10$
$\therefore q = -6$

b $f(x) = x^2+10x-6$
$f'(x) = 2x+10$
$-8 = 2a+10$
$a = -9$
when $x = -9$
$y = -15$
$\therefore b = -15$

c $\frac{dy}{dx} = 0$
$\therefore 2x+10 = 0$
$x = -5 \quad y = -31$
$(-5, -31)$

2 $\frac{dx}{dt} = 8t-8$
$\therefore -40 = 8p-8$
$-32 = 8p$
$p = -4$

$x = 4t^2-8t$
when $t = -4$
$x = 4(-4)^2-8(-4)$
$= 96$
$\therefore q = 96$

3 $y = -[4-12x+9x^2]$
$y = -4+12x-9x^2$
$\frac{dy}{dx} = 12-18x$
$6 = 12-18x$
$18x = 6$
$x = \frac{1}{3}$

$y = -(2-3(\frac{1}{3}))^2$
$y = -(1)^2$
$y = -1$
$\therefore (\frac{1}{3}, -1)$

67 Differentiating with 2 or more terms

1 (a) $\frac{dy}{dx} = 7x^6 - 3$

(b) $y = x^8 - x^7$

$\therefore \frac{dy}{dx} = 8x^7 - 7x^6$

(c) $y = 4x + x^{\frac{1}{2}} + 1$

$\frac{dy}{dx} = 4 + \frac{1}{2}x^{-\frac{1}{2}}$

or $4 + \frac{1}{2\sqrt{x}}$

(d) $y = x^{5/2} - 3x^{3/2}$

$\frac{dy}{dx} = \frac{5}{2}x^{3/2} - \frac{9}{2}x^{\frac{1}{2}}$

2 (a) $f(x) = 7x^{\frac{2}{5}} - 4x^{-1}$

$f'(x) = \frac{14}{5}x^{-3/5} + 4x^{-2}$

or $= \frac{14}{5x^{3/5}} + \frac{4}{x^2}$

(b) $f(x) = 2x^{\frac{17}{11}} - 3x^{6/11}$

$f'(x) = \frac{34}{11}x^{\frac{6}{11}} - \frac{18}{11}x^{-5/11}$

(c) $f(x) = 24 + 8x^{-1}$

$f'(x) = -8x^{-2}$

or $-\frac{8}{x^2}$

(d) $f'(x) = \frac{3}{5}x^{-\frac{6}{5}} + \frac{8}{3}x^{-\frac{2}{3}}$

3 $x = (t^1 \times t^{\frac{1}{2}}) + 10t^{-2}$

$x = t^{3/2} + 10t^{-2}$

$\frac{dx}{dt} = \frac{3}{2}t^{\frac{1}{2}} - 20t^{-3}$

$= \frac{3}{2}\sqrt{t} - \frac{20}{t^3}$ ✓

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1 (a) $y = 8x + x^{-1}$

$\frac{dy}{dx} = 8 - x^{-2}$

$0 = 8 - x^{-2}$

$0 = 8 - \frac{1}{x^2}$

$\frac{1}{x^2} = 8$

$\frac{1}{8} = x^2$

$x = \pm\frac{1}{2\sqrt{2}}$

2 $f'(x) = 6x^{-\frac{1}{4}} - x^{-\frac{1}{2}}$

or $\frac{6}{x^{\frac{1}{4}}} - \frac{1}{\sqrt{x}}$

when $x = 16$

$f'(16) = \frac{6}{16^{\frac{1}{4}}} - \frac{1}{\sqrt{16}}$

$= \frac{6}{2} - \frac{1}{4}$

$= \frac{12}{4} - \frac{1}{4}$

$= \frac{11}{4}$

3 (a) $\frac{dy}{dx} = x^2 + x - 6$

$= (x+3)(x-2)$

(b) $x = -3$ or $x = 2$

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1 $\frac{dy}{dx} = -x^{\frac{1}{2}} + x + 42$

$0 = -x^{\frac{1}{2}} + x + 42$

$x^{\frac{1}{2}} - x - 42 = 0$

$(x^{\frac{1}{2}} - 7)(x^{\frac{1}{2}} + 6) = 0$

$x^{\frac{1}{2}} = 7 \quad x^{\frac{1}{2}} \neq -6$

$\therefore x = 49 \quad y = \frac{-6517}{6}$

2 $f'(x) = 0 - \frac{1}{2}x^{-\frac{1}{2}}$

$f'(x) = 0$ for stationary point.

$0 \neq -\frac{1}{2\sqrt{x}}$ $\therefore$ no real solutions

3 $\frac{dy}{dx} = 2ax + b$

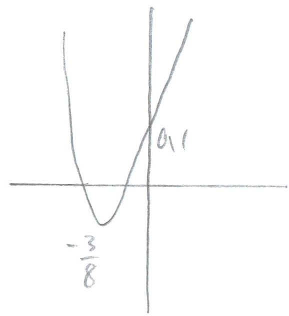
(a) • SP $0 = 2a\left(-\frac{3}{8}\right) - \frac{3}{8}(b)$

$\therefore b = -2a$ (1)

• y axis $1 = c$ (2)

• Gradient $-5 = 2a(-1) + b$

$b = 2a - 5$ (3)



(1) = (3) $-2a = 2a - 5$

$a = \frac{5}{4} \quad b = -\frac{5}{2} \quad c = 1$

(6F) Gradients, Tangents and Normals (Y1 Pure)

1 (a) $y = 4(1) + 2(1) + 1$

$y = 7$

(b) $\frac{dy}{dx} = 12x^2 + 2$

(c) when $x = 1$, $\frac{dy}{dx} = 12(1) + 2 = 14$

(d) $y - y_1 = m(x - x_1)$

$y - 7 = 14(x - 1)$

$y - 7 = 14x - 14$

$y = 14x - 7$ ✓

(e) $-\frac{1}{14}$

(f) $y - 7 = -\frac{1}{14}(x - 1)$

$14y - 98 = -x + 1$

$x + 14y = 99$ ✓

2 (a) $\frac{dy}{dx} = 12x^2 - 10x$

@ $x = 2$ $\frac{dy}{dx} = 12(4) - 10(2) = 28$

@ $x = 2$ $y = 4(8) - 5(4) + 2$

$y = 24$

$m = 28$

point $(2, 24)$

$y - 24 = 28(x - 2)$

$y = 28x - 56 + 24$

$y = 28x - 32$

(b) $m = -\frac{1}{28}$

@ $x = 3$, $y = 4(27) - 5(9) + 2$

$y = 65$

$\therefore (3, 65)$

$y - 65 = -\frac{1}{28}(x - 3)$ o.e.

3 $m = 3$

$\frac{dy}{dx} = 2x + 6$

$3 = 2x + 6$

$-3 = 2x$

$-\frac{3}{2} = x$

$y = \left(-\frac{3}{2}\right)^2 + 6\left(-\frac{3}{2}\right)$

$= \frac{9}{4} - 9 = -\frac{27}{4}$

$\therefore y + \frac{27}{4} = 3\left(x - \frac{3}{2}\right)$

$y = 3x - \frac{9}{2} - \frac{27}{4}$

$y = 3x - \frac{45}{4}$

1 (a) $y = x^{-1}$

$\frac{dy}{dx} = -x^{-2}$ or $-\frac{1}{x^2}$

when $x = 2$

$y = \frac{1}{2}$ and $\frac{dy}{dx} = -\frac{1}{4}$

$\therefore y - \frac{1}{2} = -\frac{1}{4}(x - 2)$

$y = -\frac{1}{4}x + 1$

(b) when $x = 4$

$\frac{dy}{dx} = -\frac{1}{16}$

$\therefore$ normal $m = 16$

$y - \frac{1}{4} = 16(x - 4)$

$y = 16x - 64 + \frac{1}{4}$

$y = 16x - \frac{255}{4}$

2 $\frac{dy}{dx} = 10x^4 + 1$

$11 = 10x^4 + 1$

$10x^4 = 10$

$x^4 = 1$

$x = \pm 1$

$x = 1$ as $p > 0$ $p = 1$

$\therefore$ when $x = 1$ $q = 3$

$y = 2(1) + 1 = 3$

3 when $x = -3$

$y = 9$ $\therefore (-3, 9)$

$\frac{dy}{dx} = 2x$

when $x = -3$

$\frac{dy}{dx} = -6$ Normal $m = \frac{1}{6}$

$\therefore y - 9 = \frac{1}{6}(x + 3)$

when $x = 0$

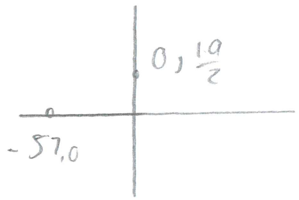
$y - 9 = \frac{1}{2}$

$y = \frac{19}{2}$

when $y = 0$

$-54 = x + 3$

$x = -57$



$\sqrt{(-57 - 0)^2 + \left(0 - \frac{19}{2}\right)^2}$

$= \frac{19\sqrt{37}}{2}$ ✓

1 $y = 2x^{3/2}$

$\frac{dy}{dx} = 3x^{\frac{1}{2}}$

m of line $= -\frac{1}{18}$

$\therefore$ normal $m = 18$

$18 = 3x^{\frac{1}{2}}$

$6 = x^{1/2}$

$36 = x$

when $x = 36$ $y = 72(6) = 432$

$y - 432 = 18(x - 36)$

$0: -432 = 18(x - 36)$

$-24 = x - 36$

$x = 12$

$\therefore 12, 0$

2 $y = -x^2 + 3x$

$\frac{dy}{dx} = -2x + 3$

when $x = 2$ $y = 2$

and $\frac{dy}{dx} = -1$

$\therefore$ m of normal $= 1$

$y - 2 = 1(x - 2)$

$y = x$

Simultaneous equations

$x = -x(x - 3)$

$x = -x^2 + 3x$

$x^2 - 2x = 0$

$x(x - 2) = 0$

$x = 0$ $x = 2$

$y = 0$ $y = 2$

68 Continued

3(a) $\frac{dy}{dx} = x^{-\frac{1}{2}} + 2$

$\frac{dy}{dx} = 6$

$\therefore 6 = \frac{1}{\sqrt{x}} + 2$

$4 = \frac{1}{\sqrt{x}}$

$\sqrt{x} = \frac{1}{4}$

$x = \frac{1}{16}$

$y = 2\sqrt{\frac{1}{16}} + 2\left(\frac{1}{16}\right) - 8$

$= \frac{1}{2} + \frac{1}{8} - 8$

$= -\frac{59}{8}$

$\therefore \left(\frac{1}{16}, -\frac{59}{8}\right)$

(b) $y + \frac{59}{8} = 6\left(x - \frac{1}{16}\right)$

$y = 0 \quad x = \frac{119}{16}$

$x = 0 \quad y = \frac{31}{4}$

$\therefore \frac{1}{2}\left(\frac{119}{16}\right)\left(\frac{31}{4}\right) = \frac{3689}{128}$

69 Increasing and Decreasing Functions

1 $f'(x) = 6x - 12$

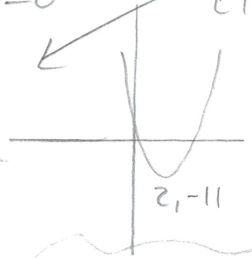
f(x) Stationary when $f'(x) = 0$

$6x - 12 = 0$

$6x = 12$

$x = 2$

Either T.P



Min point

2, -11

or

or $6x - 12 > 0$

$6x > 12$

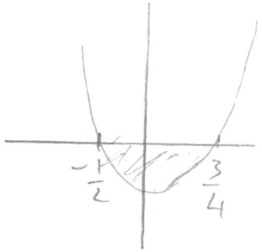
$x > 2$

2 a $f'(x) = 8x^2 - 2x - 3$

$f'(x) < 0$ for decreasing

$8x^2 - 2x - 3 < 0$

$(4x-3)(2x+1) < 0$



$-\frac{1}{2} < x < \frac{3}{4}$

1 a $f'(x) > 0$ for increasing

$f'(x) = 3ax^2 - 1$

$3ax^2 - 1 > 0$

$3(a)(4) - 1 > 0$

$12a - 1 > 0$

$a > \frac{1}{12}$

b The constant 6 differentiates to 0.

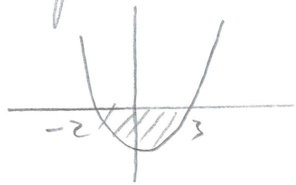
2 a $f'(x) = 2x^2 - 2x - 12$

$= 2(x^2 - x - 6)$

$= 2(x-3)(x+2)$

b Station $f'(x) = 0$ ∴ $x = 3$, $x = -2$

i decreasing $f'(x) < 0$



∴ $-2 < x < 3$

iii increas $f'(x) > 0$

∴ $x < -2$ or $x > 3$

3 $f'(x) > 0$ for increasing

$f(x) = x + x^{-1}$

$f'(x) = 1 - x^{-2}$

$f'(x) = 1 - \frac{1}{x^2}$

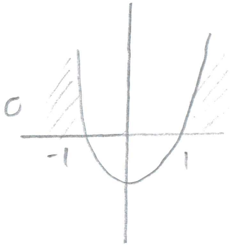
$1 - \frac{1}{x^2} > 0$

$1 > \frac{1}{x^2}$

$x^2 > 1$

$x^2 - 1 > 0$

$(x+1)(x-1) > 0$



$x < -1$ or $x > 1$

1 a $f(-\frac{1}{2}) = 2(-\frac{1}{8}) + 5(\frac{1}{4}) + 8(-\frac{1}{2}) + 3$

$= -\frac{1}{4} + \frac{5}{4} - 4 + 3$

$= 1 - 4 + 3$

$= 0$ ✓

b Polynomial Division

$(2x+1)(x^2 + 2x + 3)$

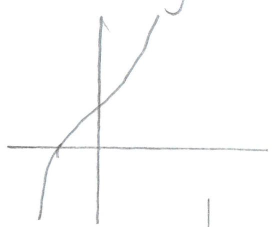
c $f'(x) = 6x^2 + 10x + 8$

d $b^2 - 4ac$: $100^2 - 4(6)(8)$

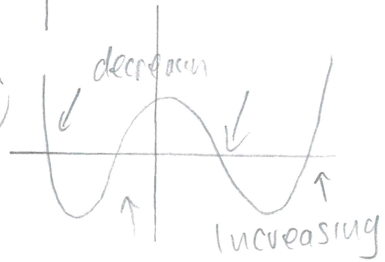
$= -92$

∴ No roots for $f'(x)$ and not stationary.

As it's a positive cubic it's always increasing.



2 a



a 2

b 2

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(10) Pure Y1

Stationary Points

(1) $f'(x)=6x^2+8x$

(b) $f'(x)=0$ for S.P

$\therefore 0=6x^2+8x$

$0=2x(3x+4)$

$x=0$ or $x=-\frac{4}{3}$ ✓

(c) when $x=0$ $f(x)=0$

$\therefore (0,0)$

when $x=-\frac{4}{3}$ $f(x)=\frac{64}{27}$

$\therefore \left(-\frac{4}{3}, \frac{64}{27}\right)$

(d) $f''(x)=12x+8$

(e) $f''(0)=8 \therefore$ min

$f''\left(-\frac{4}{3}\right)=-8 \therefore$ max

(f)

(2) (a) $\frac{dy}{dx}=20x^4$

(b) $\frac{dy}{dx}=0$ for SP.

$0=20x^4$

$x=0$

when $x=0$ $y=0$

$(0,0)$.

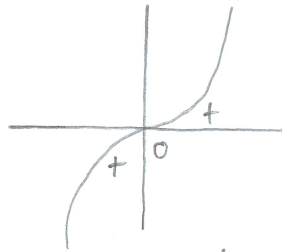
(c) when $x=-0.01$

$\frac{dy}{dx}=2\times10^{-7}$ (+)

when $x=0.01$

$\frac{dy}{dx}=2\times10^{-7}$ (+)

$\therefore$ Point of inflexion



The gradient is the same either side of the stationary point $\therefore$ point of inflexion (Point is close enough to SP)

(3) $\frac{dy}{dx}=2x^{\frac{1}{2}}-18$

for SP $\frac{dy}{dx}=0$

$\therefore 0=2\sqrt{x}-18$

$0=\sqrt{x}-9$

$9=\sqrt{x}$

$81=x$

when $x=81$

$y=\frac{4}{3}(81)^{3/2}-18(81)$

$=-486$

$\therefore (81,-486)$ is SP.

(b) $\frac{d^2y}{dx^2}=x^{-\frac{1}{2}}$

or $=\frac{1}{\sqrt{x}}$

when $x=81$

$\frac{d^2y}{dx^2}=\frac{1}{9}$

$\frac{1}{9}>0 \therefore$ min.

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(1) (a) $(x+1)(x^2-x-6)$

x^3-x^2-6x

$+x^2-x-6$

$\therefore f(x)=x^3-7x-6$

(b) $\frac{dy}{dx}=3x^2-7$

For S.P $\frac{dy}{dx}=0$

$\therefore 0=3x^2-7$

$7=3x^2$

$\frac{7}{3}=x^2$

$\pm\sqrt{\frac{7}{3}}=x$

$\pm\frac{\sqrt{21}}{3}=x$

(c) when $x=\frac{\sqrt{21}}{3}$

$y=-13.1$

when $x=-\frac{\sqrt{21}}{3}$

$y=1.13$

(d) $f''(x)=6x$

when $x=\frac{\sqrt{21}}{3}$

$f''(x)=2\sqrt{21}$

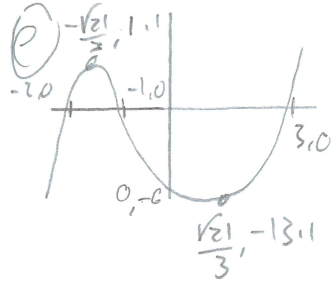
min as >0

when $x=-\frac{\sqrt{21}}{3}$

$f''(x)=-2\sqrt{21}$

max as $0<$

(e)



(2) (a) $f(2)=0$

$\therefore x=2$ is a root.

If $x^2+5x+10$ has roots $b^2-4ac\geq0$

$(5)^2-4(1)(10)=-15$

$\therefore$ no real roots.

$x=2$ is the only real root.

(b) $y=x^3+3x^2-20$

$\frac{dy}{dx}=3x^2+6x$

for SP $\frac{dy}{dx}=0$

$\therefore 0=3x^2+6x$

$0=3x(x+2)$

$x=0$ or $x=-2$

when $x=0$, $y=-20$

$(0,-20)$

when $x=-2$ $y=-16$

$(-2,-16)$

10 Contin

2 (c) $\frac{d^2y}{dx^2} = 6x+6$

(d) when $x=-2$

$\frac{d^2y}{dx^2} = -12+6$
$= -6$

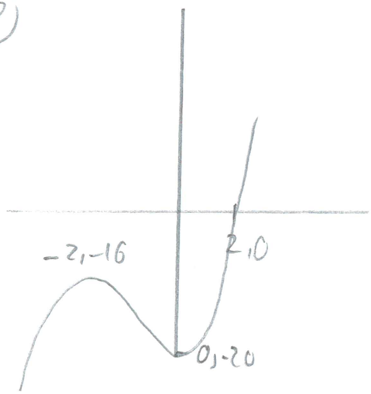
$\therefore$ max as $0 <$

when $x=0$

$\frac{d^2y}{dx^2} = 6$

$\therefore$ min as >0

(e)



1 (a) $y = \frac{x^{\frac{4}{3}} - x^1}{x^{\frac{1}{2}}}$

$y = x^{5/6} - x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{5}{6}x^{-\frac{1}{6}} - \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{6}x^{-\frac{1}{6}}\left(5 - 3x^{-\frac{1}{3}}\right)$

(b) for SP $\frac{dy}{dx} = 0$

as $x > 0$ $\quad \frac{1}{6}x^{-\frac{1}{6}} \neq 0$

$\therefore 5 - 3x^{-\frac{1}{3}} = 0$

$3x^{-\frac{1}{3}} = 5$

$x^{-1/3} = \frac{5}{3}$

$x^{\frac{1}{3}} = \frac{3}{5}$

$x = \frac{27}{125}$

(c) $\frac{d^2y}{dx^2} = -\frac{5}{36}x^{-7/6} + \frac{1}{4}x^{-\frac{3}{2}}$

when $x = \frac{27}{125}$

$\frac{d^2y}{dx^2} \approx 1.66$ $\therefore$ min as >0

2 $g'(x) = 8x^3+64$

$g'(x) = 0$ for min

$0 = 8x^3+64$

$x^3 = -8$

$x = -2$

when $x=-2$

$g(x) = -96$

$\therefore$ least value $= -96$

3 [$f'(x) > 0$ for all values.]

$f'(x) = 3x^2 - 6x + 18$

If there is a turning point $f'(x) = 0$

$0 = 3x^2 - 6x + 18$

$0 = x^2 - 2x + 6$

$b^2 - 4ac = (-2)^2 - 4(1)(6)$
$= -20$

$\therefore$ no turning points + cubic $\therefore$ gradient is always +.

f'(