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A LEVEL MATHS

YEAR 1 PURE

ANSWERS

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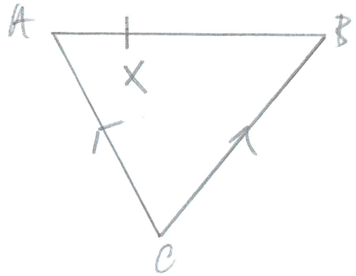
G1 Vectors. Solving Geometric Problems

(1) (a) $\vec{OA} = a$, $\vec{OB} = b$

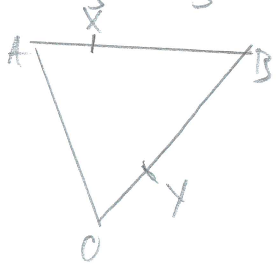
(b) $\vec{AB} = \vec{OB} - \vec{OA}$
$= b - a$

(c)



(d) $\vec{OX} = \vec{OA} + \frac{1}{3}\vec{AB}$
$= a + \frac{1}{3}(b - a)$
$= a + \frac{1}{3}b - \frac{1}{3}a$
$= \frac{2}{3}a + \frac{1}{3}b$ ///

(e)

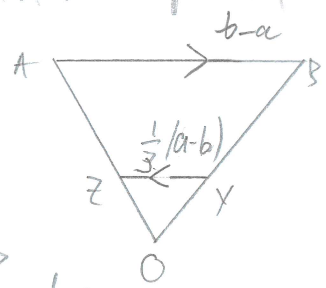


$\vec{YX} = \vec{YO} + \vec{OX}$
$= -\frac{1}{3}b + \frac{2}{3}a + \frac{1}{3}b$
$= \frac{2}{3}a$ //

$\vec{OA} = a$ //

$\therefore$ $\vec{OA}$ and $\vec{YX}$ are parallel as they are both multiples of a.

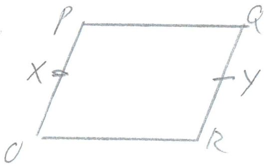
(f)



$\vec{OZ} = \frac{1}{3}a$

(11) (a) $\vec{OQ} = \vec{OP} + \vec{PQ}$
$= p + r$

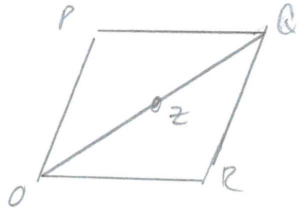
(b)



$\vec{PQ} = r$ //
$\vec{YX} = \vec{YR} + \vec{RO} + \vec{OX}$
$= -\frac{1}{2}p - r + \frac{1}{2}p$
$= -r$ //

$\therefore$ // as both $\vec{PQ}$ and $\vec{YX}$ are multiples of r.

(c)

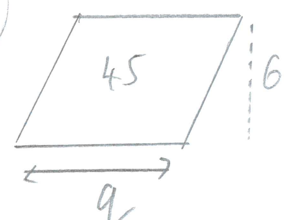


$\vec{PR} = \vec{PO} + \vec{OR}$
$= -p + r$

$\vec{PZ} = \vec{PO} + \frac{1}{2}\vec{OQ}$
$= -p + \frac{1}{2}(p + r)$
$= -p + \frac{1}{2}p + \frac{1}{2}r$
$= -\frac{1}{2}p + \frac{1}{2}r$
$= \frac{1}{2}(-p + r)$

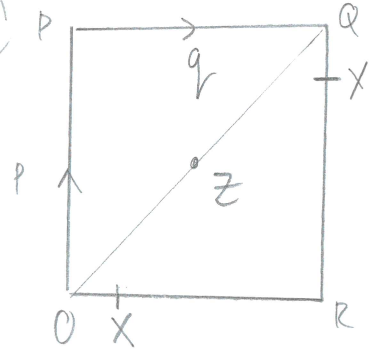
$\therefore$ // and half the length.

(d)



$6q = 45$
$q = 7.5$

(1)



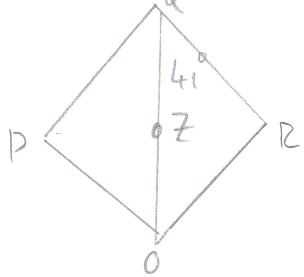
(a) $\vec{ZQ} = \frac{1}{2}\vec{OQ}$
$= \frac{1}{2}p + \frac{1}{2}q$
$= \frac{1}{2}(p + q)$

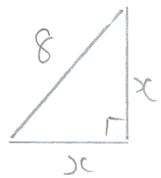
$\vec{XY} = \frac{3}{4}q + \frac{3}{4}p$
$= \frac{3}{4}(p + q)$

$\therefore$ $\vec{XY} = \frac{3}{2}\vec{ZQ}$

Ratio ZQ : XY
2 : 3

(b)

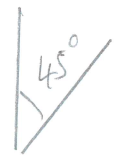




$2x^2 = 64$
$x^2 = 32$
$x = \pm 4\sqrt{2}$

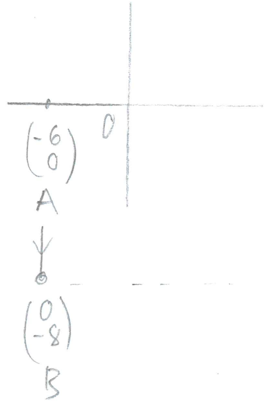
$|\vec{OR}| = 4\sqrt{2}$

(c) 45°



62 Modelling with Vectors

1



(a) $\vec{OA} = -6i$

$\vec{AB} = -8j$

(b) $\vec{OB} = -6i - 8j$

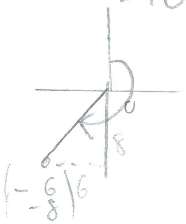
(c) $|\vec{OB}| = \sqrt{(-6)^2 + (-10)^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$= 10$

(d)



$180 + \tan^{-1}\left(\frac{6}{8}\right)$

$217°$

$\begin{pmatrix}16\\-8\end{pmatrix}$

(e) $\vec{OC} = 16i - 8j$

(f) $\vec{OB} = 10$

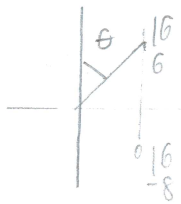
$\vec{BC} = 22$

$\vec{OC} = \sqrt{16^2 + (-8)^2}$

$= 8\sqrt{5}$

$\therefore 32 + 8\sqrt{5}$

(g)



$\tan^{-1}\left(\frac{10}{6}\right) = 059°$

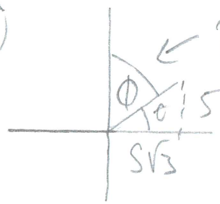
(h) $|\vec{OC}| = \sqrt{16^2 + 6^2}$

$= 2\sqrt{73}$

$\approx 17.1\text{m}$

$\frac{2\sqrt{73}}{2.4} = 7.120..$

1



Bearing

$\tan\theta = \frac{5}{5\sqrt{3}}$

$= \frac{1}{\sqrt{3}}$

$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= 30°$

$\therefore \phi = 90 - 30$

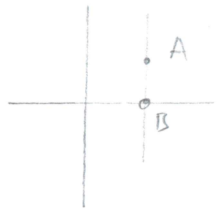
$= 60°$

(b) $|\vec{OA}| = \sqrt{(5\sqrt{3})^2 + 5^2}$

$= \sqrt{100}$

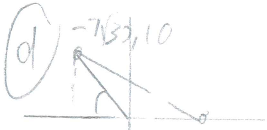
$= 10$

(c)



$\vec{OB} = 5\sqrt{3}\, i$

(d)



$\tan^{-1}\left(\frac{10}{7\sqrt{3}}\right)$

$= 39.5..$

$\therefore 270 + 39.5$

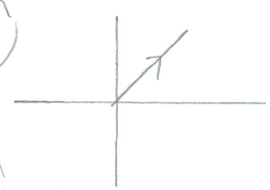
$\approx 310°$

(e) $\sqrt{(-7\sqrt{3})^2 + 10^2}$

$= \sqrt{247}$

$= 15.7$

1



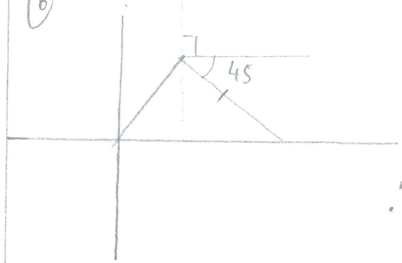
(a) $x = 20\cos 45$

$y = 20\sin 45$

$\therefore 10\sqrt{2}\, i + 10\sqrt{2}\, j$

$\begin{pmatrix}10\sqrt{2}\\10\sqrt{2}\end{pmatrix}$

(b)



$x = 10\cos 45$

$y = -10\sin 45$

$\therefore \vec{AB} = 5\sqrt{2}\, i - 5\sqrt{2}\, j$

$\vec{OB} = \vec{OA} + \vec{AB}$

$= 5\sqrt{2}\, i - 5\sqrt{2}\, j + (10\sqrt{2}\, i + 10\sqrt{2}\, j)$

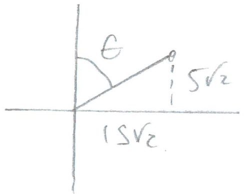
$= 15\sqrt{2}\, i + 5\sqrt{2}\, j$

$= \begin{pmatrix}15\sqrt{2}\\5\sqrt{2}\end{pmatrix}$

(c) $|\vec{OB}| = \sqrt{(15\sqrt{2})^2 + (5\sqrt{2})^2}$

$= 10\sqrt{5}$

(d) $\tan^{-1}\left(\frac{15\sqrt{2}}{5\sqrt{2}}\right) = 072°$



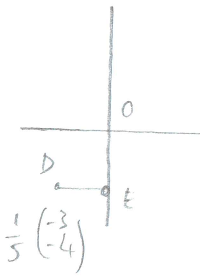
(e) $|-3i - 4j| = \sqrt{(-3)^2 + (-4)^2}$

$= 5$

$\therefore \vec{OD} = \frac{1}{5}\begin{pmatrix}-3\\-4\end{pmatrix}$

$= -\frac{3}{5}i - \frac{4}{5}j$

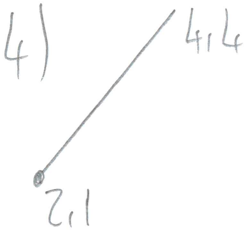
(f)



$\vec{OE} = -\frac{4}{5}j$

Year 1 63 Gradients of Curves (Differentiation)

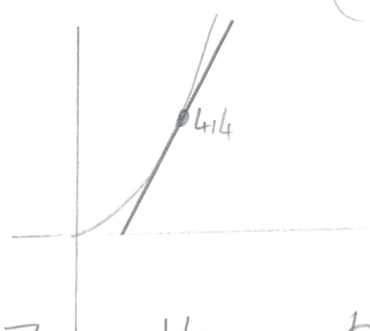
1 (a) Straight line from (2,1) to (4,4)



(b) Gradient $= \frac{y_1 - y_2}{x_1 - x_2}$

$\therefore \frac{4-1}{4-2} = \frac{3}{2}$ ✓

(c) line drawn at (4,4)



$\approx \frac{7}{4.5} \approx \frac{14}{9}$ anything sensible! with workings

1 (a) Anything $\approx \frac{6}{2}$

2 – 3 is reasonable with workings

2 Draw a tangent instead of a chord.

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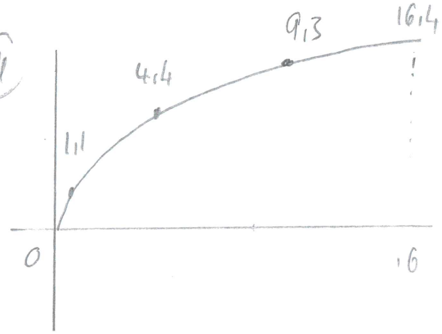
1 (a)



(b) $m \to 0$.

2 draws a line roughly at the point where $x \approx 6$

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(64) Finding the Derivative (Pure Y1)

① $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f(x) = x^2$

$\therefore f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - (x)^2}{h}$

$= \frac{\cancel{x^2} + 2hx + h^2 \cancel{- x^2}}{h}$

$= \frac{2hx + h^2}{h}$

$= 2x + h$

when $h = 0$

$f'(x) = 2x$

① $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f(x) = 4x^2 - 3x$

$\therefore f'(x) = \lim_{h \to 0} \frac{\left[4(x+h)^2 - 3(x+h)\right] - \left[4(x)^2 - 3(x)\right]}{h}$

$= \frac{4(x^2 + 2hx + h^2) - 3x - 3h - 4x^2 + 3x}{h}$

$= \frac{\cancel{4x^2} + 8hx + 4h^2 - 3h \cancel{- 4x^2}}{h}$

$= \frac{8hx + 4h^2 - 3h}{h}$

$= 8x + 4h - 3$

$= 8x - 3 + 4h$

when $h = 0$

$f'(x) = 8x - 3$

① $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f(x) = x^4$

$\therefore f'(x) = \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h}$

$= \frac{\cancel{x^4} + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 \cancel{- x^4}}{h}$

$= \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$

$= 4x^3 + 6x^2h + 4xh^2 + h^3$

when $h = 0$

$f'(x) = 4x^3$

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Y1 65 Pure

Differentiating x^n

1 (a) $4x^3$ (b) $7x^6$ (c) $6x^2$

(d) $20x^3$ (e) $\frac{3}{2}x^{\frac{1}{2}}$ (f) $-x^{-2}$

or $-\frac{1}{x^2}$ (g) $-28x^6$

(h) $2x^{-3/4}$ (i) write

as $x^{\frac{1}{2}}$ $\therefore$ $\frac{1}{2}x^{-\frac{1}{2}}$

2 (a) $f'(x) = \frac{4}{5}x^{-\frac{1}{5}}$

(b) $f'(x) = x^{-2/3}$

(c) $f(x) = 6x^{-1}$

$\therefore f'(x) - 6x^{-2}$ or $-\frac{6}{x^2}$

(d) $f'(x) = \frac{2}{5}x^{-\frac{7}{5}}$

(e) $f(x) = \frac{1}{2}x^{-2}$

$\therefore f'(x) = -x^{-3}$

3 $x = 8t^{\frac{1}{4}}$

$\frac{dx}{dt} = 2t^{-3/4}$

1 (a) $y = x(x^{\frac{1}{2}})$

$y = x^{3/2}$

$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$

(b) $y = \frac{1}{2}x^6$

$\frac{dy}{dx} = 3x^5$

(c) $y = \frac{4}{3}x^{-\frac{1}{5}}$

$\frac{dy}{dx} = -\frac{4}{15}x^{-6/5}$

or $-\frac{4}{15x^{6/5}}$

2 (a) $f(x) = 16x^{14}$

$f'(x) = 224x^{13}$

(b) $f(x) = \frac{8x}{x^{3/4}}$

$= 8x^{\frac{1}{4}}$

$\therefore f'(x) = 2x^{-3/4}$

3 $P = \frac{1}{2} \times t \times t^{1/2}$

$= \frac{1}{2}t^{3/2}$

$\therefore \frac{dP}{dt} = \frac{3}{4}t^{1/2}$

1 $h(t) = 4t^2 \times 3t^{-\frac{1}{4}}$

$= 12t^{7/4}$

$\therefore h'(t) = 21t^{3/4}$

2 $y = 2t^{1/4}$

$\frac{dy}{dt} = \frac{1}{2}t^{-3/4}$

when $t = 16$

$\frac{dy}{dt} = \frac{1}{2}(16)^{-3/4}$

$= \frac{1}{2} \times \left(\frac{1}{8}\right)$

$= \frac{1}{16}$

3 $f'(x) = 4x$

$\therefore 4x = 64$

$x = 16$

Steve Blades