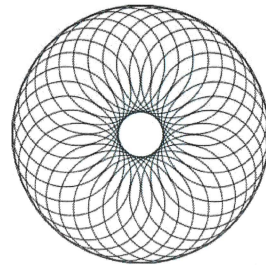


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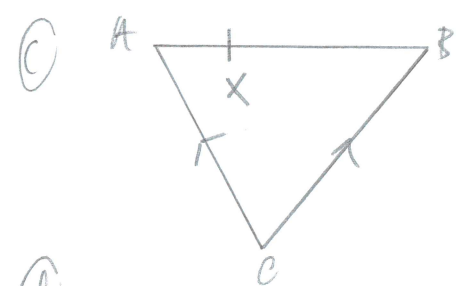
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<ul style="list-style-type: none"><li>(1) Indices</li><li>(2) Expanding Brackets</li><li>(3) Factorising Expressions</li><li>(4) More Indices (Negative and Fractional)</li><li>(5) Working with Surds</li><li>(6) Solving Quadratic Equations</li><li>(7) Completing the Square for Quadratics Expressions</li><li>(8) Function Notation</li><li>(9) Sketching Quadratic Graphs</li><li>(10) The Discriminant for Quadratic Equations</li><li>(11) Applications of Quadratics Equations</li><li>(12) Solving Linear Simultaneous Equations</li><li>(13) Linear &amp; Non-Linear Simultaneous Equations</li><li>(14) Graphing Simultaneous Equations</li><li>(15) Linear Inequalities</li><li>(16) Quadratic Inequalities</li><li>(17) Graphing Inequalities</li><li>(18) Shading Inequalities</li><li>(19) Cubic Graphs</li><li>(20) Quartic Graphs</li><li>(21) Reciprocal Graphs</li><li>(22) The Intersection of Graphs</li><li>(23) Transforming Graphs (Translations)</li><li>(24) Transforming Graphs (Stretching/Reflecting)</li><li>(25) Straight Line Graphs in the form <math>y = mx + c</math></li><li>(26) More Straight Line Graphs</li><li>(27) Straight Line Graphs (Parallel &amp; Perpendicular)</li><li>(28) The Geometry of Straight Lines</li><li>(29) The Application of Linear Graphs</li><li>(30) Circle Geometry Midpoint &amp; Perpendicular</li></ul>	<ul style="list-style-type: none"><li>(31) The Equation of a Circle</li><li>(32) Circles and Straight Lines (Intersections)</li><li>(33) Circles (Tangents and Chords)</li><li>(34) Circles and Triangles</li><li>(35) Algebraic Fractions</li><li>(36) Polynomial Division</li><li>(37) The Factor and Remainder Theorem</li><li>(38) An Introduction to Mathematical Proof</li><li>(39) Methods of Proof</li><li>(40) Binomial Expansion (Using Pascal's Triangle)</li><li>(41) Binomial Expansion (Factorial Notation)</li><li>(42) Binomial Expansion (The <math>\binom{n}{r}</math> Method)</li><li>(43) Binomial Expansion (Problem Solving)</li><li>(44) Binomial Expansion (Estimations and Approximations)</li><li>(45) The Cosine Rule</li><li>(46) The Sine Rule</li><li>(47) Areas of a Triangles</li><li>(48) Triangles (Problem Solving)</li><li>(49) Sine, Cosine &amp; Tangent Graphs</li><li>(50) Transforming Graphs (Trigonometry)</li><li>(51) The 'CAST' Diagram for Trig Ratios</li><li>(52) Trigonometry (Exact Values)</li><li>(53) Proving Trigonometric Identities</li><li>(54) Solving Basic Trigonometric Equations</li><li>(55) More Challenging Trigonometric Equations</li><li>(56) Using Identities to Solve Trig Equations</li><li>(57) Vectors (Introduction)</li></ul>	<ul style="list-style-type: none"><li>(58) Vector Notation (Column and i and j form)</li><li>(59) Vectors (Magnitude and Direction)</li><li>(60) Vectors (Position and Direction Vectors)</li><li>(61) Vector Geometry</li><li>(62) Application of Vectors</li><li>(63) Differentiation (Gradients of Curves)</li><li>(64) Differentiation from 1st Principles</li><li>(65) Differentiating <math>x^n</math> (Basic Powers of )</li><li>(66) Differentiation (Quadratic Expression)</li><li>(67) Differentiation (Multiple Terms)</li><li>(68) Differentiation (Gradients, Tangents and Normals)</li><li>(69) Differentiation (Increasing and Decreasing Functions)</li><li>(70) Differentiation (Stationary Points)</li><li>(71) Differentiation (Gradient Functions)</li><li>(72) The Applications of Differentiation</li><li>(73) Integration (Basic Expressions (<math>x^n</math>))</li><li>(74) Indefinite Integrals</li><li>(75) Integration (Finding <math>c</math> and Finding Functions)</li><li>(76) Integration (Definite Integrals)</li><li>(77) Integration (Basic Areas Under Curves)</li><li>(78) Integration ('Negative and Positive Areas')</li><li>(79) Integration (Areas between Curves and Lines)</li><li>(80) Basic Exponential Functions</li><li>(81) 'The' Exponential Function <math>y = e^x</math></li><li>(82) Applications of Basic Exponential Models</li><li>(83) Logarithms (Simplifying &amp; Evaluating)</li><li>(84) Logarithms (The Log Laws)</li><li>(85) Logarithms (Log and Exponential Equations)</li></ul>
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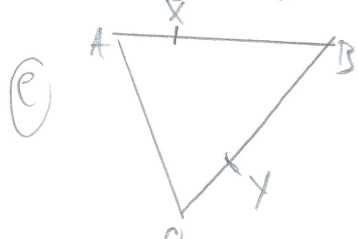
# 61 Vectors Solving Geometric Problems

① a)  $\vec{OA} = a, \vec{OB} = b$

b)  $\vec{AB} = \vec{OB} - \vec{OA} = b - a$



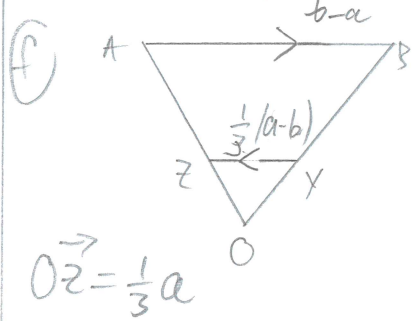
d)  $\vec{OX} = \vec{OA} + \frac{1}{3}\vec{AB}$   
 $= a + \frac{1}{3}(b - a)$   
 $= a + \frac{1}{3}b - \frac{1}{3}a$   
 $= \frac{2}{3}a + \frac{1}{3}b //$



$\vec{YX} = \vec{YO} + \vec{OX}$   
 $= -\frac{1}{3}b + \frac{2}{3}a + \frac{1}{3}b$   
 $= \frac{2}{3}a //$

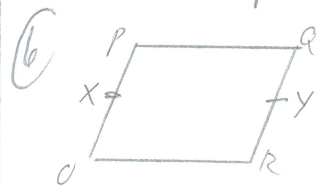
$\vec{OA} = a //$

$\therefore \vec{OA}$  and  $\vec{YX}$  are parallel as they are both multiples of  $a$ .



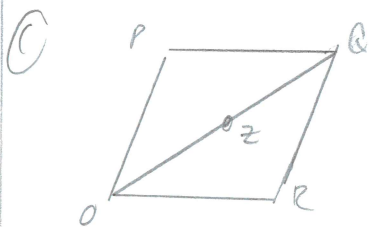
$\vec{OZ} = \frac{1}{3}a$

① a)  $\vec{OQ} = \vec{OP} + \vec{PQ} = p + r$



$\vec{PQ} = r //$   
 $\vec{YX} = \vec{YR} + \vec{RO} + \vec{OX}$   
 $= -\frac{1}{2}r - r + \frac{1}{2}r = -r //$

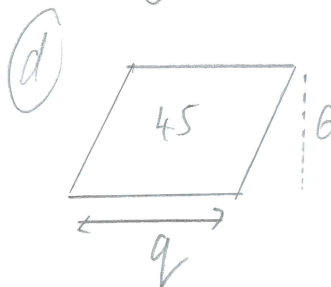
$\therefore //$  as both  $\vec{PQ}$  and  $\vec{YX}$  are multiples of  $r$ .



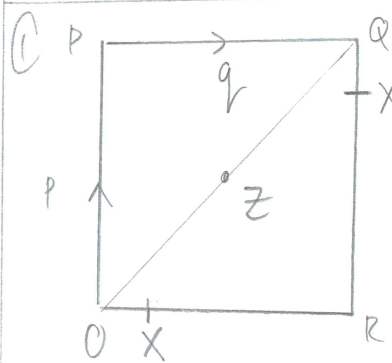
$\vec{PR} = \vec{PO} + \vec{OR} = -p + r$

$\vec{PZ} = \vec{PO} + \frac{1}{2}\vec{OR}$   
 $= -p + \frac{1}{2}(pr)$   
 $= -p + \frac{1}{2}p + \frac{1}{2}r = -\frac{1}{2}p + \frac{1}{2}r = \frac{1}{2}(-p + r)$

$\therefore //$  and half the length.



$6q = 45$   
 $q = 7.5$

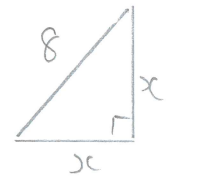
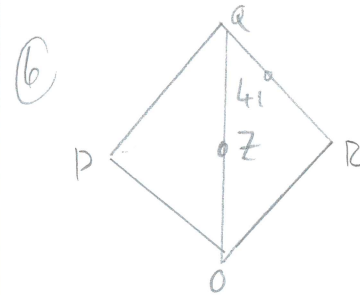


① a)  $\vec{ZQ} = \frac{1}{2}\vec{OQ} = \frac{1}{2}p + \frac{1}{2}q = \frac{1}{2}(p + q)$

$\vec{XY} = \frac{3}{4}q + \frac{3}{4}p = \frac{3}{4}(p + q)$

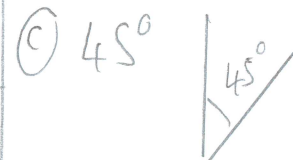
$\therefore \vec{XY} = \frac{3}{2}\vec{ZQ}$

Ratio  $ZQ : XY = 2 : 3$

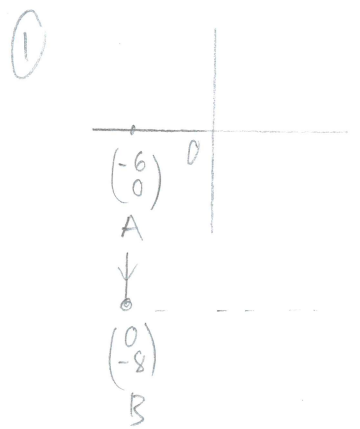


$2x^2 = 64$   
 $x^2 = 32$   
 $x = \pm 4\sqrt{2}$

$|\vec{OR}| = 4\sqrt{2}$



# 62 Modelling with vectors

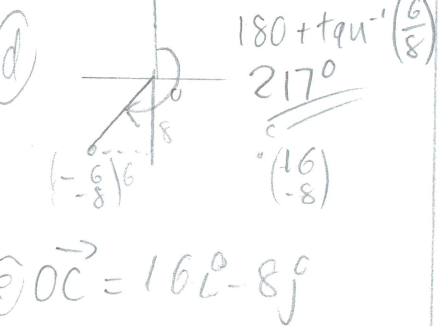


①

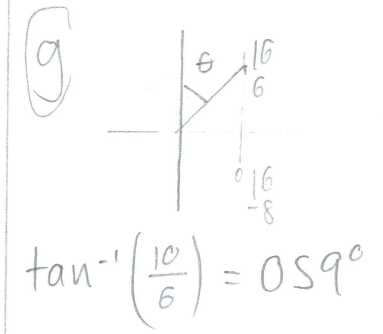
a)  $\vec{OA} = -6\mathbf{i}$   
 $\vec{AB} = -8\mathbf{j}$

b)  $\vec{OB} = -6\mathbf{i} - 8\mathbf{j}$

c)  $|\vec{OB}| = \sqrt{(-6)^2 + (-8)^2}$   
 $= \sqrt{36 + 64}$   
 $= \sqrt{100}$   
 $= 10$

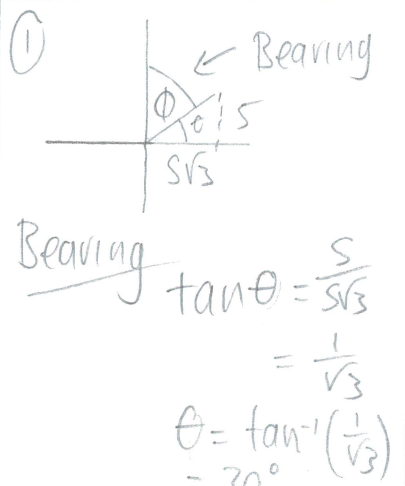


f)  $|\vec{OB}| = 10$   
 $|\vec{BC}| = 22$   
 $|\vec{OC}| = \sqrt{16^2 + (-8)^2}$   
 $= 8\sqrt{5}$   
 $\therefore 32 + 8\sqrt{5}$



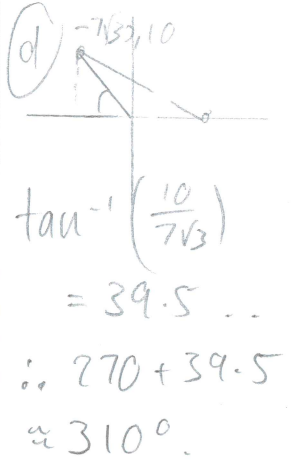
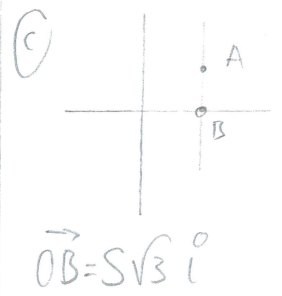
h)  $|\vec{OC}| = \sqrt{16^2 + 6^2}$   
 $= 2\sqrt{73}$   
 $\approx 17.1\text{m}$

$\frac{2\sqrt{73}}{2.4} = 7.120\dots$

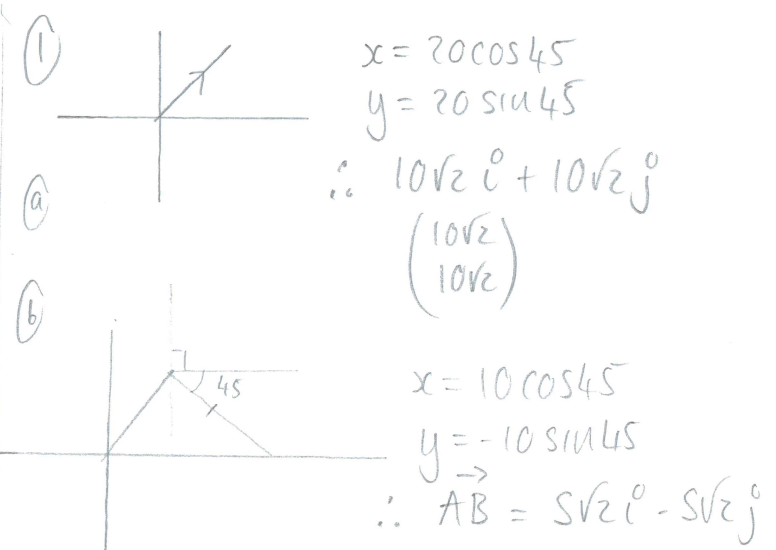


a.  $\phi = 90 - 30$   
 $= 60^\circ$

b)  $|\vec{OA}| = \sqrt{(5\sqrt{3})^2 + 5^2}$   
 $= \sqrt{100}$   
 $= 10$



e)  $\sqrt{(-7\sqrt{3})^2 + 10^2}$   
 $= \sqrt{247}$   
 $= 15.7$

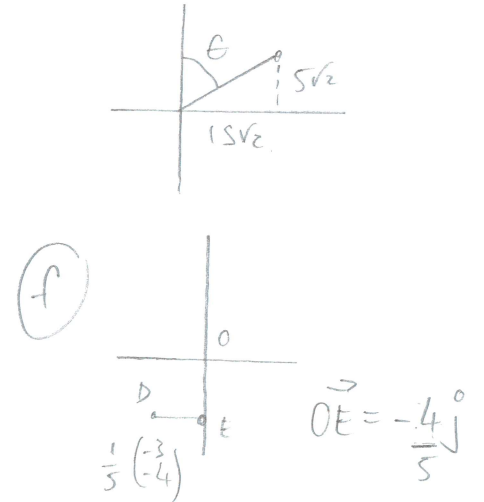


$\vec{OB} = \vec{OA} + \vec{AB}$   
 $= 5\sqrt{2}\mathbf{i} - 5\sqrt{2}\mathbf{j} + (10\sqrt{2}\mathbf{i} + 10\sqrt{2}\mathbf{j})$   
 $= 15\sqrt{2}\mathbf{i} + 5\sqrt{2}\mathbf{j}$   
 $= (15\sqrt{2})\mathbf{i} + (5\sqrt{2})\mathbf{j}$

c)  $|\vec{OB}| = \sqrt{(15\sqrt{2})^2 + (5\sqrt{2})^2}$   
 $= 10\sqrt{5}$

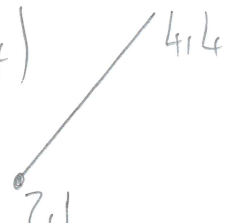
d)  $\tan^{-1}\left(\frac{15\sqrt{2}}{5\sqrt{2}}\right) = 0.72^\circ$

e)  $|-3\mathbf{i} - 4\mathbf{j}| = \sqrt{(-3)^2 + (-4)^2}$   
 $= 5$   
 $\therefore \vec{OD} = \frac{1}{5} \begin{pmatrix} -3 \\ -4 \end{pmatrix}$   
 $= -\frac{3}{5}\mathbf{i} - \frac{4}{5}\mathbf{j}$



Year 1 **63** Gradients  
of Curves (Differentiation)

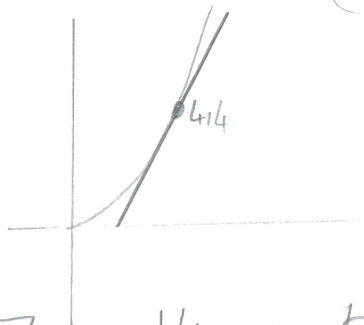
1) a) Straight line from  
(2,1) to (4,4)



b) Gradient =  $\frac{y_1 - y_2}{x_1 - x_2}$

$\therefore \frac{4-1}{4-2} = \frac{3}{2} \checkmark$

c) line drawn at (4,4)

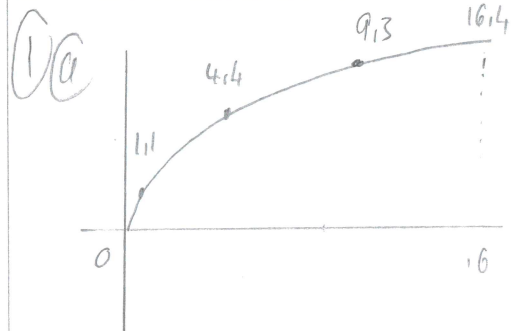


$\approx \frac{7}{4.5} \approx \frac{14}{9}$  anything  
sensible! with workings

1) a) Anything  $\approx \frac{6}{2}$

2-3 is reasonable  
with workings

2) Draw a tangent  
instead of a chord.



b)  $m \rightarrow 0$ .

2) draws a line  
roughly at the  
point where  $x \approx 6$

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## (64) Finding the Derivative (Pure Y1)

$$\textcircled{1} f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = x^2$$
$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 - (x)^2}{h}$$

$$= \frac{x^2 + 2hx + h^2 - x^2}{h}$$

$$= \frac{2hx + h^2}{h}$$

$$= 2x + h$$

when  $h=0$

$$\underline{\underline{f'(x) = 2x}}$$

$$\textcircled{1} f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = 4x^2 - 3x$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{[4(x+h)^2 - 3(x+h)] - [4(x)^2 - 3(x)]}{h}$$

$$= \frac{4(x^2 + 2hx + h^2) - 3x - 3h - 4x^2 + 3x}{h}$$

$$= \frac{4x^2 + 8hx + 4h^2 - 3h - 4x^2}{h}$$

$$= \frac{8hx + 4h^2 - 3h}{h}$$

$$= 8x + 4h - 3$$

$$= 8x - 3 + 4h$$

when  $h=0$

$$\underline{\underline{f'(x) = 8x - 3}}$$

$$\textcircled{1} f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = x^4$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^4 - x^4}{h}$$

$$= \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h}$$

$$= \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$$

$$= 4x^3 + 6x^2h + 4xh^2 + h^3$$

when  $h=0$

$$\underline{\underline{f'(x) = 4x^3}}$$

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Y1 (65) Pure  
Differentiating  $x^n$

1) (a)  $4x^3$  (b)  $7x^8$  (c)  $6x^2$

(d)  $20x^3$  (e)  $\frac{3}{2}x^{\frac{1}{2}}$  (f)  $-x^2$   
or  $-\frac{1}{x^2}$  (g)  $-28x^6$

(h)  $2x^{-3/4}$  (i) Write  
as  $x^{\frac{1}{2}}$   $\therefore \frac{1}{2}x^{-\frac{1}{2}}$

(2) (a)  $f'(x) = \frac{4}{5}x^{-\frac{1}{5}}$

(b)  $f'(x) = x^{-2/3}$

(c)  $f(x) = 6x^{-1}$   
 $\therefore f'(x) = 6x^{-2}$  or  $-\frac{6}{x^2}$

(d)  $f''(x) = \frac{2}{5}x^{-\frac{7}{5}}$

(e)  $f(x) = \frac{1}{2}x^{-2}$   
 $\therefore f'(x) = -x^{-3}$

(3)  $x = 8t^{\frac{1}{4}}$   
 $\frac{dx}{dt} = 2t^{-3/4}$

(1) (a)  $y = x(x^{\frac{1}{2}})$   
 $y = x^{3/2}$

$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$

(b)  $y = \frac{1}{2}x^6$

$\frac{dy}{dx} = 3x^5$

(c)  $y = \frac{4}{3}x^{-\frac{1}{5}}$

$\frac{dy}{dx} = -\frac{4}{15}x^{-\frac{6}{5}}$

or  $-\frac{4}{15x^{6/5}}$

(2) (a)  $f(x) = 16x^{14}$

$f'(x) = 224x^{13}$

(b)  $f(x) = \frac{8x}{x^{3/4}}$

$= 8x^{\frac{1}{4}}$

$\therefore f'(x) = 2x^{-3/4}$

(3)  $P = \frac{1}{2}xtxt^{1/2}$   
 $= \frac{1}{2}t^{3/2}$

$\therefore \frac{dP}{dt} = \frac{3}{4}t^{1/2}$

(1)  $h(t) = 4t^2 \times 3t^{-\frac{1}{4}}$   
 $= 12t^{7/4}$

$\therefore h'(t) = 21t^{3/4}$

(2)  $y = 2t^{1/4}$

$\frac{dy}{dt} = \frac{1}{2}t^{-3/4}$

when  $t = 16$

$\frac{dy}{dt} = \frac{1}{2}(16)^{-3/4}$

$= \frac{1}{2} \times \left(\frac{1}{8}\right)$   
 $= \frac{1}{16}$

(3)  $f''(x) = 4x$

$\therefore 4x = 64$

$x = 16$

Steve Blades