WWW.M4THS.COM

A LEVEL MATHS

YEAR 1 PURE

ANSWERS

ONLY AVAILABLE FROM

WWW.M4THS.COM

(1) Indices
(2) Expanding Brackets
(3) Factorising Expressions
(4) More Indices (Negative and Fractional)
(5) Working with Surds
(6) Solving Quadratic Equations
(7) Completing the Square for Quadratics Expressions
(8) Function Notation
(9) Sketching Quadratic Graphs
(10) The Discriminant for Quadratic Equations
(11) Applications of Quadratics Equations
(12) Solving Linear Simultaneous Equations
(13) Linear & Non-Linear Simultaneous Equations
(14) Graphing Simultaneous Equations
(15) Linear Inequalities
(16) Quadratic Inequalities
(17) Graphing Inequalities
(18) Shading Inequalities
(19) Cubic Graphs
(20) Quartic Graphs
(21) Reciprocal Graphs
(22) The Intersection of Graphs
(23) Transforming Graphs (Translations)
(24) Transforming Graphs (Stretching/Reflecting)
(25) Straight Line Graphs in the form $y = mx + c$
(26) More Straight Line Graphs
(27) Straight Line Graphs (Parallel & Perpendicular)
(28) The Geometry of Straight Lines
(29) The Application of Linear Graphs
(30) Circle Geometry Midpoint & Perpendicular
(31) The Equation of a Circle
(32) Circles and Straight Lines (Intersections)
(33) Circles (Tangents and Chords)
(34) Circles and Triangles
(35) Algebraic Fractions
(36) Polynomial Division
(37) The Factor and Remainder Theorem
(38) An Introduction to Mathematical Proof
(39) Methods of Proof
(40) Binomial Expansion (Using Pascal's Triangle)
(41) Binomial Expansion (Factorial Notation)
(42) Binomial Expansion (The $\binom{n}{r}$Method)
(43) Binomial Expansion (Problem Solving)
(44) Binomial Expansion (Estimations and Approximations)
(45) The Cosine Rule
(46) The Sine Rule
(47) Areas of a Triangles
(48) Triangles (Problem Solving)
(49) Sine, Cosine & Tangent Graphs
(50) Transforming Graphs (Trigonometry)
(51) The 'CAST' Diagram for Trig Ratios
(52) Trigonometry (Exact Values)
(53) Proving Trigonometric Identities
(54) Solving Basic Trigonometric Equations
(55) More Challenging Trigonometric Equations
(56) Using Identities to Solve Trig Equations
(57) Vectors (Introduction)
(58) Vector Notation (Column and i and j form)
(59) Vectors (Magnitude and Direction)
(60) Vectors (Position and Direction Vectors)
(61) Vector Geometry
(62) Application of Vectors
(63) Differentiation (Gradients of Curves)
(64) Differentiation from 1st Principles
(65) Differentiating x^n (Basic Powers of)
(66) Differentiation (Quadratic Expression)
(67) Differentiation (Multiple Terms)
(68) Differentiation (Gradients, Tangents and Normals)
(69) Differentiation (Increasing and Decreasing Functions)
(70) Differentiation (Stationary Points)
(71) Differentiation (Gradient Functions)
(72) The Applications of Differentiation
(73) Integration (Basic Expressions (x^n))
(74) Indefinite Integrals
(75) Integration (Finding c and Finding Functions)
(76) Integration (Definite Integrals)
(77) Integration (Basic Areas Under Curves)
(78) Integration ('Negative and Positive Areas')
(79) Integration (Areas between Curves and Lines)
(80) Basic Exponential Functions
(81) 'The' Exponential Function $y = e^x$
(82) Applications of Basic Exponential Models
(83) Logarithms (Simplifying & Evaluating)
(84) Logarithms (The Log Laws)
(85) Logarithms (Log and Exponential Equations)

5G Equations and Identities

① $\frac{\sin x}{\cos x} \equiv \tan x$

$\therefore \sin x = \cos x$

$\frac{\sin x}{\cos x} = 1$

$\tan x = 1$

$x = \tan^{-1}(1)$

$x = 45 \pm 180n$

$x = 45, 225$

② $\cos^2 x \equiv 1 - \sin^2 x$

ⓐ $\therefore (1-\sin^2 x) + \sin x \equiv 1$

$\cancel{1} - \sin^2 x + \sin x = \cancel{1}$

$0 = \sin^2 x - \sin x$

$0 = \sin x(\sin x - 1)$

ⓑ $\sin x(\sin x - 1) = 0$

$\sin x = 0$

$x = 0 \pm 360n$

$x = 180 \pm 360n$

$x = 0, 360$

$x = 180$

$\sin x = 1$

$x = \sin^{-1}(1)$

$x = 90 \pm 360n$

$x = 90 \pm 360n$

$x = 90$

③ ⓐ $(2\sin x + 1)(\sin x - 3) = 0$

ⓑ $(2\sin x + 1)(\sin x - 3) = 0$

$2\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$

$x = \sin^{-1}\left(-\frac{1}{2}\right)$

$x = -30 \pm 360n$

$x = -150 \pm 360n$

$x = 330$

$x = 210$

$\sin x - 3 = 0$

$\sin x \neq 3$

No Solutions

① ⓐ $\cos^2 x + \sin^2 x = 1$

$\therefore \sin^2 x = 1 - \cos^2 x$

$8(1-\cos^2 x) - 10\cos x - 1 = 0$

$8 - 8\cos^2 x - 10\cos x - 1 = 0$

$-8\cos^2 x - 10\cos x + 7 = 0$

$8\cos^2 x + 10\cos x - 7 = 0$

$(4\cos x + 7)(2\cos x - 1) = 0$

ⓑ $4\cos x + 7 = 0$

$\cos x \neq -\frac{7}{4}$

$\therefore$ no real Solutions

$2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$

$x = \cos^{-1}\left(\frac{1}{2}\right)$

$x = 60 \pm 360n$

$x = 300 \pm 360n$

$x = 60, 300$

② $(3\tan A + 1)(\tan A - 1) = 0$

$3\tan A + 1 = 0$

$\tan A = -\frac{1}{3}$

$A = \tan^{-1}\left(-\frac{1}{3}\right)$

$A = -18.4 \pm 180n$

$A = -18.4, 161.6$

$\tan A - 1 = 0$

$\tan A = 1$

$A = \tan^{-1}(1)$

$A = 45 \pm 180n$

$A = 45, -135$

③ $\frac{\sin x}{\cos x} = 2\sin x$

$\sin x = 2\sin x\cos x$

$0 = 2\sin x\cos x - \sin x$

$0 = \sin x(2\cos x - 1)$

$\sin x = 0$

$x = 0, 180, 360$

$2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$

$x = 60, 300$

① ⓐ either $\theta - 20 = \theta$ ①

or $\theta - 20 = 180 - \theta$ ②

by symmetry

① has no solutions

$\therefore \theta - 20 = 180 - \theta \pm 360n$ ②

$2\theta = 200 \pm 360n$

$\theta = 100 \pm 180n$

$\theta = 100, 280$

ⓑ $1 + \sin^2 4\theta = 2\sin 4\theta$

$\sin^2 4\theta - 2\sin 4\theta + 1 = 0$

$(\sin 4\theta - 1)^2 = 0$

$\sin 4\theta = 1$

$4\theta = \sin^{-1}(1)$

$4\theta = 90° \pm 360n$

$\theta = 22.5 \pm 90n$

$\theta = 22.5, 112.5, -67.5, -157.5$

② Discriminant won't work here

$8(1-\cos^2 x) - 22\cos x - 23 = 0$

$8\cos^2 x + 22\cos x + 15 = 0$

Solving $\cos x \neq -\frac{5}{4}, \cos x \neq -\frac{3}{2}$

No real solutions $\therefore$

③ LHS we know that

$\cos^2 A + \sin^2 A = 1$

$\cos^2 A + 1 + 2\sin A + \sin^2 A$

$\cos^2 A + \sin^2 A + 1 + 2\sin A$

S.6 Continued.

3 continued

(a) $1 + 1 + 2\sin A$

$2 + 2\sin A$

$2(1+\sin A)$ Q.E.D.

(b) $2(1+\sin 2x) = 0$

$1 + \sin 2x = 0$

$\sin 2x = -1$

$2x = \sin^{-1}(-1)$

$2x = -90 \pm 360n$

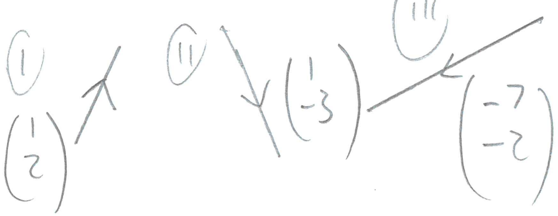
$2x = -90 \pm 360n$

$x = -45 \pm 180n$

$x = -45, 135$

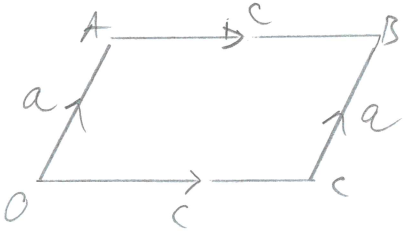
Pure Y1 57 Vectors

(1) (i) (ii) (iii)



(The column vectors above give the resultant vector)

(2) (a)



(i) $\vec{OB} = \vec{OA} + \vec{AB}$
$= a + c$

(ii) $\vec{AX} = \frac{1}{2}\vec{AB}$
$= \frac{1}{2}c$

(iii) $\vec{AY} = \vec{AO} + \frac{1}{2}\vec{OC}$
$= -a + \frac{1}{2}c$ or $\frac{1}{2}c - a$

(b) $\vec{XY} = \vec{XB} + \vec{BC} + \vec{CY}$
$= \frac{1}{2}c - a - \frac{1}{2}c$
$= -a$

$\vec{OA} = a$

∴ parallel

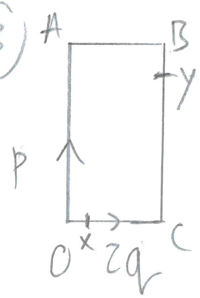
(1) $\frac{9}{2} = 4.5$

$\therefore 6 \times 4.5 = 27$

$p = 27$

(2) (i) ✓ (iii) ✗ (v) ✓
(ii) ✗ (iv) ✓

(3)

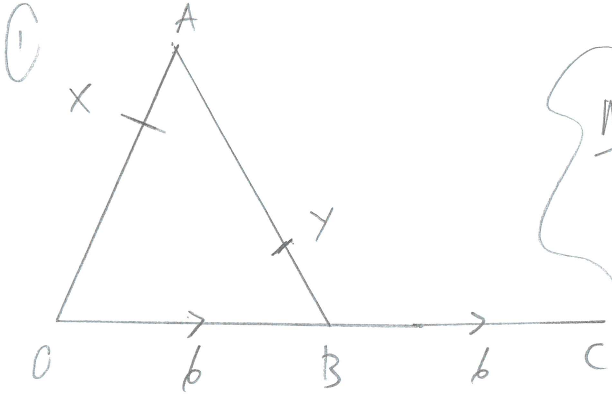


$\vec{OB} = \vec{OA} + \vec{AB}$
$= p + 2q$

$\vec{XY} = \vec{XC} + \frac{3}{4}\vec{CB}$
$= \frac{3}{4}(2q) + \frac{3}{4}(p)$
$= \frac{3}{4}(2q + p)$
$= \frac{3}{4}(p + 2q)$

∴ $\vec{OB}$ and $\vec{XY}$ are parallel as both multiples of $p+2q$

(1)



NB You can pick many ways to do 1a

(a) If XYC is a straight line $\vec{XY}$ and $\vec{YC}$ will be parallel.

$\vec{XY} = \vec{XA} + \vec{AY}$
$= \vec{XA} + \frac{2}{3}\vec{AB}$
$= \frac{1}{3}a + \frac{2}{3}(b - a)$
$= \frac{2}{3}b - \frac{1}{3}a$
or $\frac{1}{3}(2b - a)$

$\vec{YC} = \vec{YB} + \vec{BC}$
$= \frac{1}{3}\vec{AB} + \vec{BC}$
$= \frac{1}{3}(b - a) + b$
$= \frac{4}{3}b - \frac{1}{3}a$
$= \frac{1}{3}(4b - a)$

∴ not parallel as $2b-a$ and $4b-a$ are different.

(b) $\vec{OD} = \vec{OY} + \lambda\vec{XY}$
$= \vec{OB} + \vec{BY} + \lambda\left[\frac{1}{3}(2b - a)\right]$
$= \vec{OB} + \frac{1}{3}\vec{BA} + \lambda\left[\frac{1}{3}(2b - a)\right]$
$= b + \frac{1}{3}(a - b) + \lambda\left[\frac{2}{3}b - \frac{1}{3}a\right]$
$= \frac{1}{3}a + \frac{2}{3}b + \lambda\left[\frac{2}{3}b - \frac{1}{3}a\right]$

λ is a scalar multiple

let $\lambda = 1$ for example
$\therefore \frac{4}{3}b$.

58 Representing vectors

1. (i) $5i+3j$ (ii) $-3i-7j$
(iii) $14i+11j$ (iv) $4i-8j$
(v) $-8i-10j$ (vi) $-9i-21j$

2. (i) $\begin{pmatrix}2\\-2\end{pmatrix}$ (ii) $\begin{pmatrix}6\\15\end{pmatrix}$ (iii) $\begin{pmatrix}4\\8\end{pmatrix}$
(iv) $\begin{pmatrix}0\\-12\end{pmatrix}$ (v) $\begin{pmatrix}3\\-6\end{pmatrix}$ (vi) $\begin{pmatrix}10\\35\end{pmatrix}$

3. $\vec{AB} = p+7q-(2p-3q)$
$= -p+10q$
($\vec{AB}$ is just $\vec{OB}-\vec{OA}$)

---

1. i's $3p+8=-10$
$3p=-18$
$p=-6$
j's $18+2q=4$
$2q=-14$
$q=-7$

2. $\vec{AB} = \vec{OB}-\vec{OA}$
$\therefore 5p-3q = \vec{OB}-(9p+2q)$
$5p-3q+(9p+2q) = \vec{OB}$
$14p-q = \vec{OB}$

3. $\frac{p}{5} = \frac{-4}{-12}$
$p = \frac{1}{3}\times 5$
$p = \frac{5}{3}$

---

1. (a) $\begin{pmatrix}1\\-2\end{pmatrix}+\begin{pmatrix}p\\2p\end{pmatrix}=\lambda\begin{pmatrix}4\\5\end{pmatrix}$
Simultaneous Equations
$1+p=4\lambda$ ①
$-2+2p=5\lambda$ ②
$5+5p=20\lambda$ ①
$-8+8p=20\lambda$ ②
① − ② $13-3p=0$
$p=\frac{13}{3}$

(b) When $p=\frac{13}{3}$

$a+b=\begin{pmatrix}1+\frac{13}{3}\\-2+\frac{26}{3}\end{pmatrix}$
$=\begin{pmatrix}\frac{16}{3}\\\frac{20}{3}\end{pmatrix}$
$=\frac{1}{3}\begin{pmatrix}16\\20\end{pmatrix}$
$\therefore c$ as $a+b=\frac{1}{3}c$

2. $\begin{pmatrix}4\\6\end{pmatrix}+\mu\begin{pmatrix}-4\\10\end{pmatrix}=\lambda\begin{pmatrix}1\\1\end{pmatrix}$
Simultaneous Equations
$4-4\mu=\lambda$ ①
$6+10\mu=\lambda$ ②
① = ② $\therefore 4-4\mu=6+10\mu$
$-2=14\mu$
$-\frac{1}{7}=\mu$

59 Magnitude and Direction (Vectors)

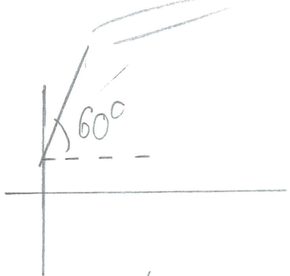
1 a) $|a| = \sqrt{2^2+3^2}$
$= \sqrt{4+9}$
$= \sqrt{13}$

b) $a+b = \begin{pmatrix}2\\3\end{pmatrix} + \begin{pmatrix}0\\-4\end{pmatrix} = \begin{pmatrix}2\\-1\end{pmatrix}$

or $2i - j$

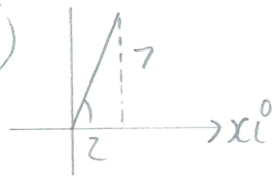
c) $|a+b| = \sqrt{2^2+(-1)^2}$
$= \sqrt{5}$

2



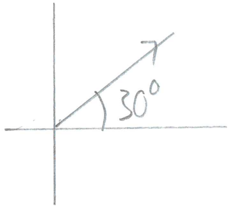
$xi = 5\cos(60) = 5 \times \frac{1}{2} i$
$yj = 5\sin(60) = 5 \times \frac{\sqrt{3}}{2} j$
$\therefore \frac{5}{2} i + \frac{5\sqrt{3}}{2} j$

3



$\tan^{-1}\left(\frac{7}{2}\right) = 74.1°$

1



$x = 8\cos(30) i$
$= 8\left(\frac{\sqrt{3}}{2}\right) i$
$= 4\sqrt{3} i$

$y = 8\sin(30) j$
$= 8\left(\frac{1}{2}\right) j$
$= 4j$

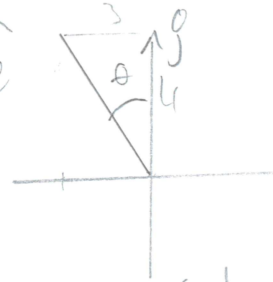
$\therefore 4\sqrt{3} i + 4j$

2 $\frac{1}{\sqrt{(-3)^2+4^2}} (-3i + 4j)$

$\frac{1}{5}(-3i + 4j)$

a) $-\frac{3}{5} i + \frac{4}{5} j$

6

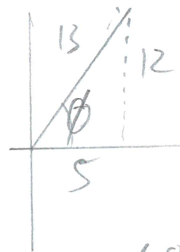


$\theta = \tan^{-1}\left(\frac{3}{4}\right)$

$\theta = 36.9°$ or $323.1°$

3 $(1)^2 + (p)^2 = (5\sqrt{2})^2$
$1 + p^2 = 50$
$p^2 = 49$
$p = \pm 7$

1



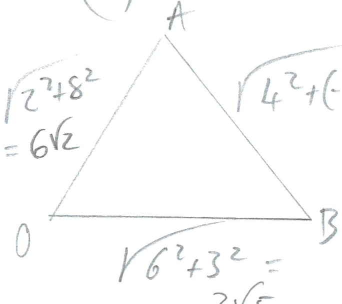
$x: 4\cos Ø i$
$y = 4\sin Ø j$
$x = 4\left(\frac{5}{13}\right) i$
$y = 4\left(\frac{12}{13}\right) j$

$\therefore \frac{20}{13} i$

2 $\vec{AB} = \vec{OB} - \vec{OA}$
$= \begin{pmatrix}6\\3\end{pmatrix} - \begin{pmatrix}2\\8\end{pmatrix}$

a) $= \begin{pmatrix}4\\-5\end{pmatrix}$ or $4i - 5j$

b)



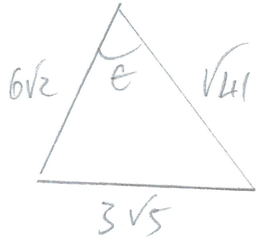
$\sqrt{2^2+8^2} = 6\sqrt{2}$

$\sqrt{4^2+(-5)^2} = \sqrt{41}$

$\sqrt{6^2+3^2} = 3\sqrt{5}$

$P = \sqrt{41} + 3\sqrt{5} + 6\sqrt{2}$
$= 21.357...$
$\therefore 21.4$

c)

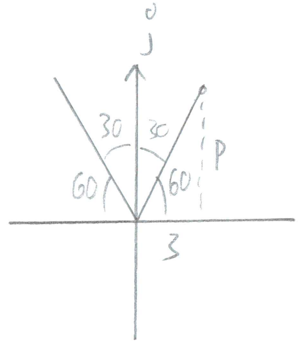


$\theta = \cos^{-1}\left(\frac{(6\sqrt{2})^2 + (\sqrt{41})^2 - (3\sqrt{5})^2}{2(6\sqrt{2})(\sqrt{41})}\right)$

$\theta = 51.26...$

$\therefore A = \frac{1}{2} \times 6\sqrt{2} \times \sqrt{41} \times \sin\theta$
$= 21.189...$
$= 21.2u^2$

3



$\tan 60 = \frac{P}{3}$
$3\sqrt{3} = p$

60 Position Vectors

1 $\vec{OA} = 3i + 7j$

a $\vec{OB} = 4i - 8j$

b $\vec{AB} = \vec{OB} - \vec{OA}$

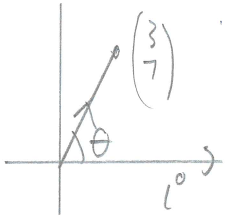
$= i - 15j$

c $\vec{BA} = -\vec{AB} \therefore -i + 15j$

d $|\vec{AB}| = \sqrt{(-1)^2 + (15)^2}$

$= \sqrt{226}$

e



$\tan^{-1}\left(\frac{7}{3}\right)$

$= 66.8°$

2 a $\vec{OA} = 8i + 5j$

$\vec{OB} = 7i + j$

b $\vec{BA} = \vec{OA} - \vec{OB}$

$= \begin{pmatrix}8\\5\end{pmatrix} - \begin{pmatrix}7\\1\end{pmatrix}$

$= \begin{pmatrix}1\\4\end{pmatrix}$

c $\sqrt{1^2 + 4^2} = \sqrt{17}$

3 $\vec{CD} = \vec{OD} - \vec{OC}$

$\begin{pmatrix}2\\6\end{pmatrix} = \begin{pmatrix}p\\q\end{pmatrix} - \begin{pmatrix}-4\\8\end{pmatrix}$

$\begin{pmatrix}-2\\14\end{pmatrix} = \begin{pmatrix}p\\q\end{pmatrix}$

1 $\vec{AB} = \vec{OB} - \vec{OA}$

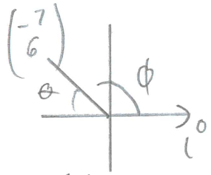
a $\begin{pmatrix}-3\\-2\end{pmatrix} = \vec{OB} - \begin{pmatrix}-4\\8\end{pmatrix}$

$\begin{pmatrix}-7\\6\end{pmatrix} = \vec{OB}$

$\therefore |\vec{OB}| = \sqrt{(-7)^2 + 6^2}$

$= \sqrt{85}$

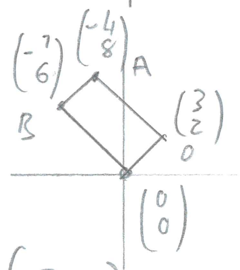
b



$180 - \tan^{-1}\left(\frac{6}{7}\right)$

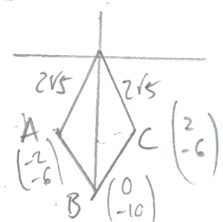
$= 139.4°$

c



$C\begin{pmatrix}3, 2\end{pmatrix}$

2



a $\vec{OC} = 2i - 6j$

b $\frac{1}{2} \times 10 \times 12 = 72$

3 $\vec{DC} = \vec{OC} - \vec{OD}$

$\begin{pmatrix}8\\-3\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix} - \begin{pmatrix}p\\q\end{pmatrix}$

$\begin{pmatrix}p\\q\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix} - \begin{pmatrix}8\\-3\end{pmatrix}$

$\begin{pmatrix}p\\q\end{pmatrix} = \begin{pmatrix}-6\\4\end{pmatrix}$

$\therefore |\vec{OD}| = \sqrt{(-6)^2 + 4^2}$

$= 2\sqrt{13}$

1 $(6m)^2 + (8m)^2 = 25$

$36m^2 + 64m^2 = 25$

$100m^2 = 25$

$m^2 = \frac{1}{4}$

$m = \pm\frac{1}{2}$

when $m = \frac{1}{2}$

$\vec{OP} = \begin{pmatrix}3\\4\end{pmatrix}$

when $m = -\frac{1}{2}$

$\vec{OP} = \begin{pmatrix}-3\\-4\end{pmatrix}$

$\therefore (3, 4)$ or $(-3, -4)$

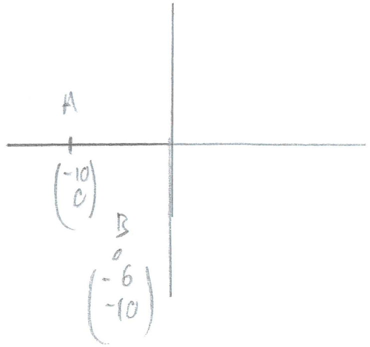
2 a $|\vec{OA}| = \sqrt{(-10)^2}$

$= 10$

$|\vec{OB}| = \sqrt{(-6)^2 + (-10)^2}$

$= \sqrt{136}$

$= 2\sqrt{34}$



b For isosceles triangle one pair of vectors must have the same modulus.

$\vec{AB} = \vec{OB} - \vec{OA}$

$= \begin{pmatrix}-6\\-10\end{pmatrix} - \begin{pmatrix}-10\\0\end{pmatrix}$

$= \begin{pmatrix}4\\-10\end{pmatrix}$

$\therefore |\vec{AB}| = \sqrt{4^2 + (-10)^2}$

$= \sqrt{116}$

$= 2\sqrt{24}$

$\therefore |\vec{AB}| \neq |\vec{OB}|$, $\therefore |\vec{AB}| \neq |\vec{OA}|$

and $|\vec{OA}| = |\vec{OB}|$ (any of these).

c $\frac{1}{2} \times 10 \times 10 = 50$

3 $\vec{AB} = \vec{OB} - \vec{OA}$

$q\begin{pmatrix}0\\1\end{pmatrix} = p\begin{pmatrix}1\\0\end{pmatrix} - \begin{pmatrix}5\\-6\end{pmatrix}$

$\therefore 0 = p - 5$ and $q = 6$

$\therefore \vec{AB} = \begin{pmatrix}0\\6\end{pmatrix}$ or $6j$