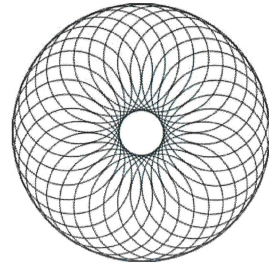


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Pure 46 The Sine Rule.

① $\frac{y}{\sin 62} = \frac{6}{\sin 26}$

$y = \frac{6 \sin 62}{\sin 26}$

$y = 12.1 \text{ cm}$

② $\frac{\sin y}{16} = \frac{\sin 40}{14}$

$y = \sin^{-1}\left(\frac{16 \sin 40}{14}\right)$

$y = 47.3^\circ$

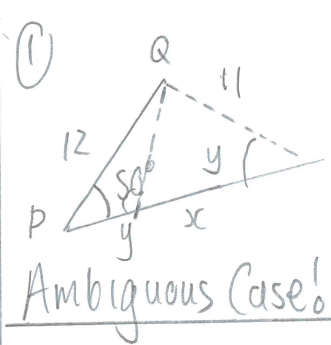
③ let $\angle ACB = y$

$\frac{\sin y}{13} = \frac{\sin 41}{12}$

$y = \sin^{-1}\left(\frac{13 \sin 41}{12}\right)$

$y = 45.3^\circ$

$\therefore \angle CBD = 41 + y = 86.3^\circ$



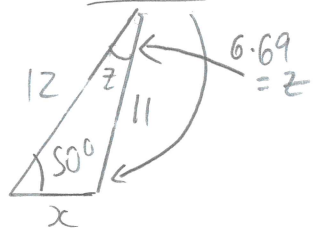
Ambiguous Case!

$\frac{\sin y}{12} = \frac{\sin 50}{11}$

$\sin y = \frac{12 \sin 50}{11}$

$\sin y = 0.835$

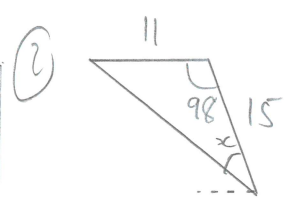
$\therefore y = 56.7^\circ \text{ or } 123.3^\circ$



$\therefore \frac{x}{\sin(6.68^\circ)} = \frac{11}{\sin 50}$

$x = \frac{11 \sin(6.68^\circ)}{\sin 50}$

$x = 1.67 \checkmark$



$\angle CDA = 82^\circ$
as ABCD is a trapezium.

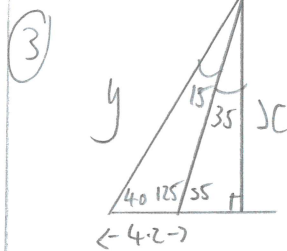
$\therefore \angle BDA = 82 - x$

$\frac{\sin x}{11} = \frac{\sin 98}{15}$

$x = \sin^{-1}\left(\frac{11 \sin 98}{15}\right)$

$x = 46.56^\circ$

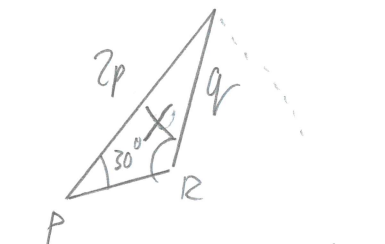
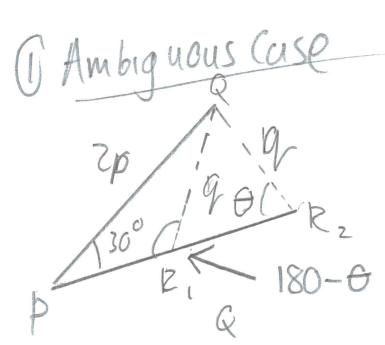
$\therefore \angle BDA = 82 - 46.56 = 35.4^\circ$



$\frac{y}{\sin 125} = \frac{15}{\sin 40}$

$y = 13.292$

$\therefore x = \frac{13.292 \times x}{\sin 40} = 8.54$



We know that $\sin \theta = \sin(180 - \theta)$

$\angle QPR = 180 - \theta$
as it's obtuse

$\therefore \frac{\sin X}{2p} = \frac{\sin 30}{q}$

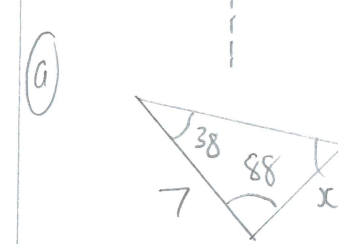
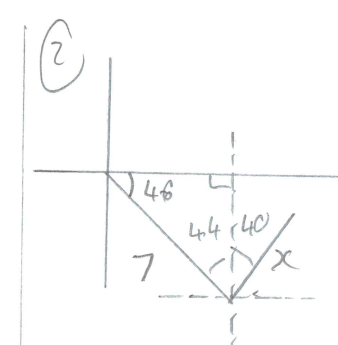
$\frac{\sin X}{2p} = \frac{1/2}{q}$

$\sin X = \frac{1/2 \times 2p}{q}$

$\sin X = \frac{p}{q}$

$X = \arcsin\left(\frac{p}{q}\right)$

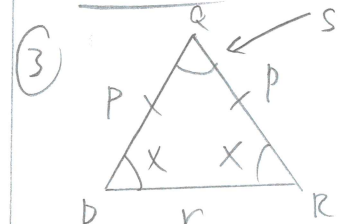
$\therefore \angle QRP = 180 - \arcsin\left(\frac{p}{q}\right)$
as it's obtuse.



$\frac{x}{\sin 38} = \frac{7}{\sin 54}$
 $x = 5.33$

⑥ $\sqrt{7^2 + 5.33^2 - 2(7)(5.33) \cos 88}$

$= 8.65$



Perimeter = $2p + r$

We need r in terms of p.

$x = \angle QPR$

$\frac{r}{\sin 120} = \frac{p}{\sin x}$

$\frac{r}{1/4} = \frac{p}{\sin \angle QPR}$

(46) Continued

$$r = \frac{\frac{1}{4}P}{\sin \angle QPR}$$

$$r = \frac{P}{4 \sin \angle QPR}$$

$\therefore 2P + \frac{P}{4 \sin \angle QPR}$ is the perimeter

Factoring $\frac{P}{4}$ out

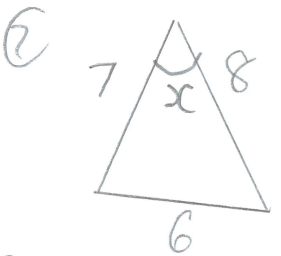
$$\frac{P}{4} \left[8 + \frac{1}{\sin \angle QPR} \right] \text{ Q.E.D.}$$

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47 Area of Triangles

① $\frac{1}{2}absinC$

$A = \frac{1}{2} \times 13 \times 11 \times \sin 34$
 $A = 39.9822 \dots$
 $\therefore 40.0 \text{ cm}^2$



② Cosine Rule

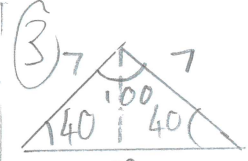
$\cos x = \frac{7^2 + 8^2 - 6^2}{2(7)(8)}$

$x = 46.6^\circ$ to 1dp
 N.B. you can find 57.9° or 75.5°

⑥ Using the diagram above.

$A = \frac{1}{2} \times 7 \times 8 \times \sin 46.6$
 $= 20.3 \text{ cm}^2$ to 3SF.

N.B You can use other Angles and sides!



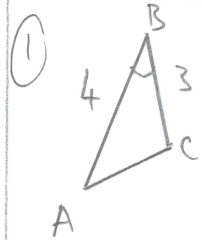
Using Cosine Rule

$x^2 = 7^2 + 7^2 - 2(7)(7)\cos 100$
 $x = 10.724$

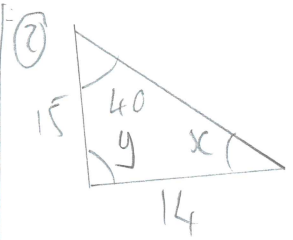
Using $\frac{1}{2}absinC$

$\frac{1}{2} \times 7 \times 10.724 \times \sin 40$
 $= 24.1277899 \dots$
 $\therefore 24.1 \text{ cm}^2$ to 3SF.

N.B you can use other methods!



① Area = $\frac{1}{2}absinC$
 $= \frac{1}{2} \times 4 \times 3 \times \sin(\angle B)$
 $= 6 \times (\frac{1}{8})$
 $= \frac{6}{8}$
 $= \frac{3}{4}$ Q.E.D

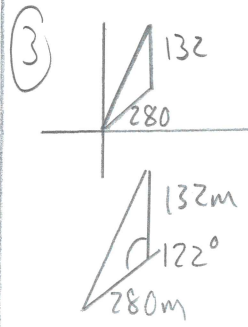


$\frac{\sin x}{15} = \frac{\sin 40}{14}$

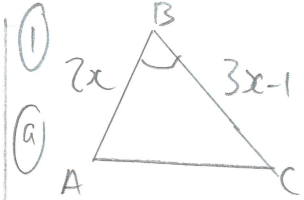
$x = \sin^{-1}(\frac{15 \sin 40}{14})$
 $x = 43.52 \dots$

$\therefore y = 180 - 40 - 43.52$
 $y = 96.472 \dots$

Area = $\frac{1}{2}absinC$
 $\text{Area} = \frac{1}{2} \times 15 \times 14 \times \sin(96.47)$
 $\text{Area} = 104 \text{ cm}^2$

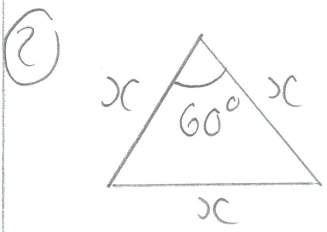


③ $A = \frac{1}{2} \times 280 \times 132 \times \sin 122$
 $A = 15671.93 \text{ m}^2$



① Area = $\frac{1}{2}absinC$
 $\therefore 0.8 = \frac{1}{2} \times (2x) \times (3x-1) \times 0.4$
 $0.8 = 0.2(2x)(3x-1)$
 $4 = 6x^2 - 2x$
 $0 = 6x^2 - 2x - 4$
 $0 = 3x^2 - x - 2$
 $0 = (3x+2)(x-1)$

⑥ $x = 1$ as $x = -\frac{2}{3}$
 When $x = 1$, $2x = 2$ and $3x-1 = 2$
 $\therefore AB = BC$ which makes it isosceles.

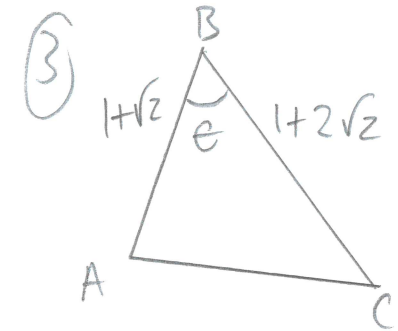


② $A = \frac{1}{2}x^2 \sin 60$
 $3\sqrt{3} = \frac{1}{2}x^2(\frac{\sqrt{3}}{2})$
 $3\sqrt{3} = \frac{\sqrt{3}}{4}x^2$

$12 = x^2$
 $\pm \sqrt{12} = x$
 $\pm 2\sqrt{3} = x$

$\sin 60 = \frac{\sqrt{3}}{2}$
 When $x = 2\sqrt{3}$
 Perimeter = $3x$
 $\therefore 3(2\sqrt{3}) = 6\sqrt{3}$

47 Continued



$$\text{Area} = \frac{1}{2}ab\sin C$$

$$\therefore \frac{1}{2} = \frac{1}{2}(1+\sqrt{2})(1+2\sqrt{2})\sin \theta$$

$$1 = (1+2\sqrt{2}+\sqrt{2}+4)\sin \theta$$

$$1 = (5+3\sqrt{2})\sin \theta$$

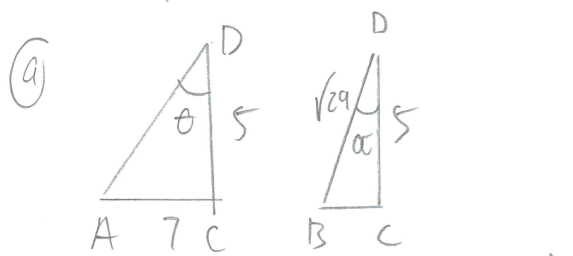
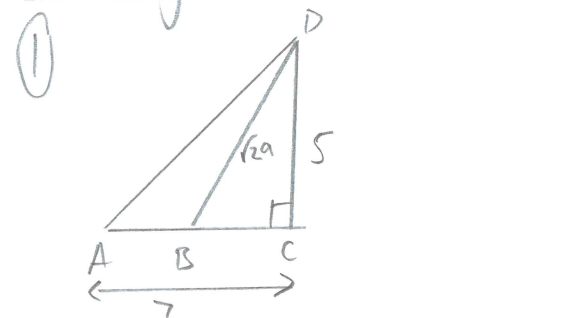
$$\frac{1}{5+3\sqrt{2}} = \sin \theta$$

$$\frac{1(S-3\sqrt{2})}{(S+3\sqrt{2})(S-3\sqrt{2})} = \sin \theta$$

$$\frac{S-3\sqrt{2}}{25-18\sqrt{2}+18\sqrt{2}-18} = \sin \theta$$

$$\frac{S-3\sqrt{2}}{7} = \sin \theta \quad \checkmark$$

(48) Pure Solving Triangle Problems



$\theta = \tan^{-1}\left(\frac{7}{5}\right)$ $\alpha = \cos^{-1}\left(\frac{5}{\sqrt{29}}\right)$
 $\angle ADB = \theta - \alpha$
 $= 54.46 - 21.8$
 $= 32.7^\circ$

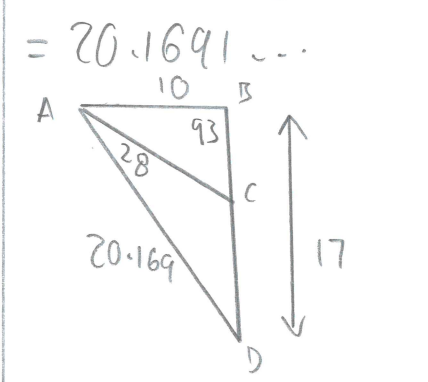
(b) $\triangle ADC$ Area = $\frac{7 \times 5}{2}$
 $= 17.5$

$\triangle BDC = \frac{1}{2} \times \sqrt{29} \times 5 \times \sin 21.8$
 $= 12.5$

$\triangle ADB = 17.5 - 12.5$
 $= 5$

(c) $AD = \sqrt{5^2 + 7^2}$
 $= \sqrt{74}$
 $BD = \sqrt{29}$
 $AB = 7 - \sqrt{5^2 + 29}$
 $= 7 - \sqrt{34}$
 $\therefore \sqrt{74} + \sqrt{29} + (7 - \sqrt{34})$
 $= 15.2 \text{ cm}$

(1) (a) $AD = \sqrt{10^2 + 17^2 - 2(10)(17)\cos 93}$
 $= 20.1691 \dots$



$\angle DAB$
 $\frac{\sin x}{17} = \frac{\sin 93}{20.169 \dots}$
 $x = 57.3$
 $\therefore \angle BAC = 57.3 - 28$
 $= 29.32$

(and $\angle BDA = 29.678$)

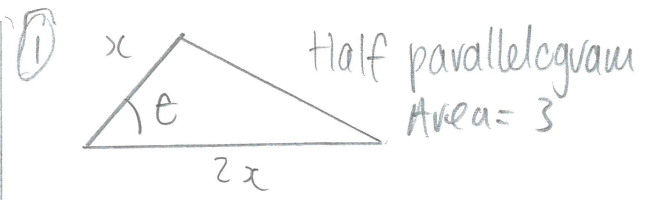
AC length
 $\frac{AC}{\sin 93} = \frac{10}{\sin 57.68}$
 $AC = 11.817 \dots$
 $= 11.8$

(b) Dark Triangle
 Area = $\frac{1}{2}(11.8)(10)\sin 29.32$
 $= 28.933 \dots$

Full Triangle
 Area = $\frac{1}{2}(10)(17)\sin 93$
 $= 84.883 \dots$

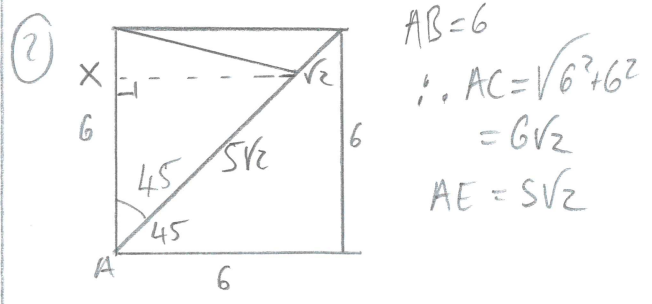
\therefore Proportion = $\frac{28.933}{84.88}$
 $\approx 34\%$

(c) $ABC = 10 + 11.817 \dots$
 $+ \frac{\sqrt{10^2 + 11.817^2 - 2(10)(11.817)\cos(29.32)}}{\cos(29.32)}$
 $= 10 + 11.817 + 5.79$
 $= 27.6 \text{ cm}$



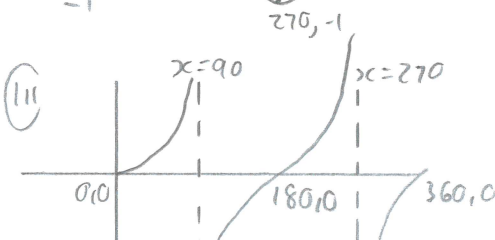
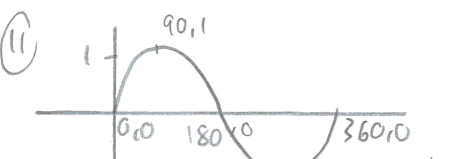
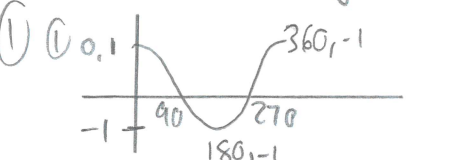
$\frac{1}{2}(x)(2x)\sin \theta = 3$
 $2x^2 \sin \theta = 6$
 $\sin \theta = \frac{3}{x^2}$

We know $0 < \sin \theta \leq 1$
 as $0 < \theta \leq 180$ and $x > 0$
 $\therefore 0 < \frac{3}{x^2} \leq 1$
 $0 < 3 \leq x^2$
 $\therefore x \geq +\sqrt{3}$

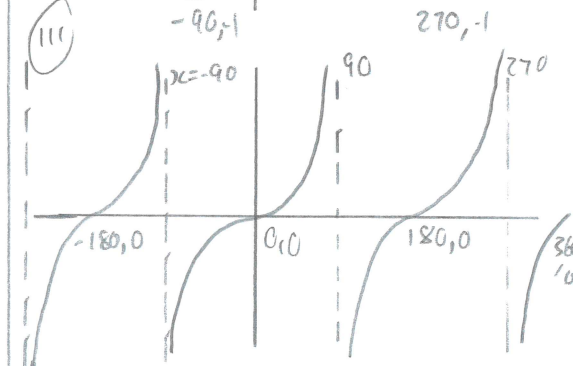
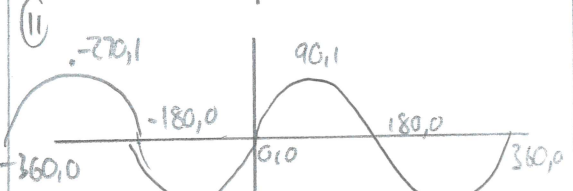
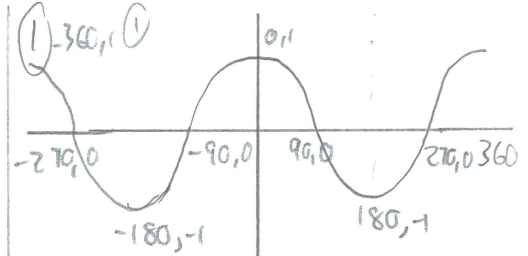


$AB = 6$
 $\therefore AC = \sqrt{6^2 + 6^2}$
 $= 6\sqrt{2}$
 $AE = 5\sqrt{2}$
 XE and AX are perpendicular
 $XE = 5\sqrt{2} \sin 45 = 5$
 $\therefore \triangle ABE$ area = $\frac{5 \times 6}{2} = 15$
 Proportion = $\frac{15}{36}$
 $= \frac{5}{12}$

49 Graphs of Sine Cosine and Tangent



- 2) 360°
 360°
 180°
- 3) $\sin(0) = 0$ not 2
 Sin has a max of 1 and min of -1, not 2 and -2.



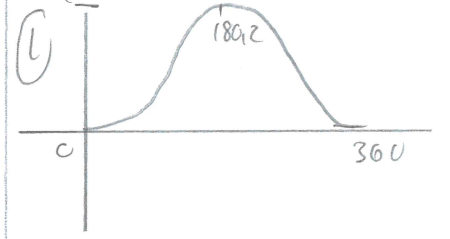
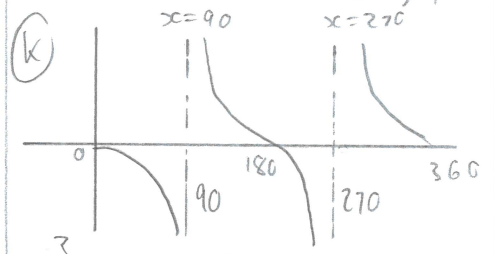
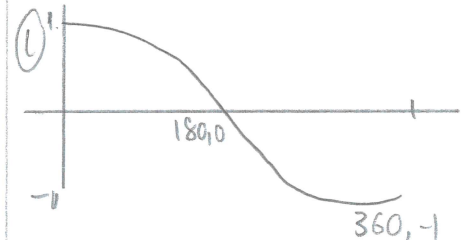
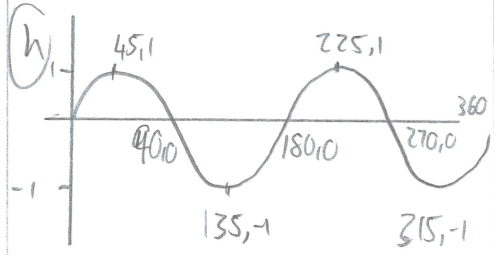
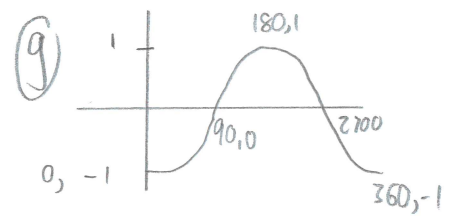
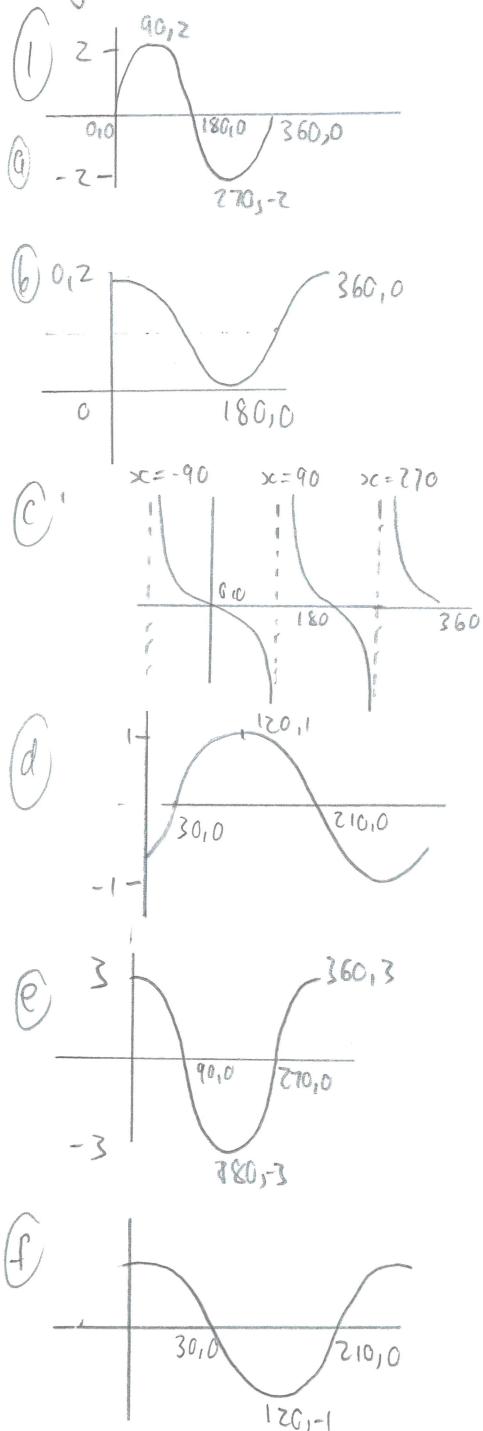
- 2) a) $180 - 60 = 120^\circ$
 b) $360 - 90 = 270^\circ$
 c) $45 + 180n \therefore 45, 225, 405$
- 3) a) $x=0, x=-180, x=180$
 b) $x=-90, x=90$
 c) None.

- 1) 1 ✓
 2 x
 3 ✓
 4 ✓
- 5 ✓
 6 ✓
 7 ✓
 8 ✓
- 9 ✓
 10 ✓
 11 x
 12 ✓

- 2) a) $135^\circ, 315^\circ$
 b) $240^\circ, 320^\circ$
 c) $45^\circ, 315^\circ$
 d) $95.7^\circ, 264.3^\circ$
 e) $90^\circ, 270^\circ$
 f) Never!

- 3) a) 2
 b) 2
 c) 3

(50) Transforming Trigonometric Graphs

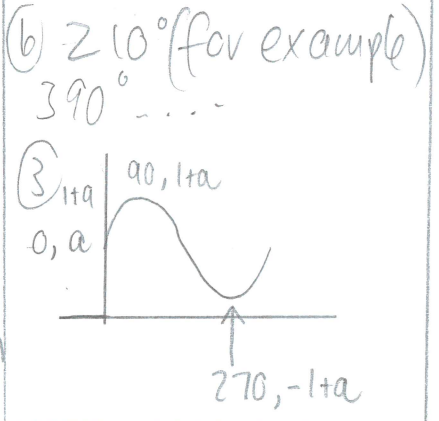


(2) $\cos x$ has a minimum value of -1 . A translation of $\begin{pmatrix} 0 \\ k \end{pmatrix}$ where $k > 1$ will translate the graph

move than 1 unit in the positive y direction
 \therefore no longer crossing the x axis.

(1) $y = k \cos x$
 (a) $\sqrt{2} = k \cos(360)$
 $\sqrt{2} = k(1)$
 $\therefore k = \sqrt{2}$

(b) $(180, -\sqrt{2})$
 (2) $y = \tan x$ has an asymptote at $x = 90^\circ$
 $(x-a)$ is a translation of $\begin{pmatrix} a \\ 0 \end{pmatrix} \therefore 90 + 30 = 120$.



(1) (a) $a = 3$
 (b) $0 < b < \frac{1}{2}$

- (2) (a) $a = 2, b = 225$
 (b) $a = -2, b = 45$
 (c) $a = 2, b = 135$
 (d) $a = -2, b = 225$
 (3) $1 \leq k < 3$

All examples

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 Steve Blades