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YEAR 1 PURE

ANSWERS

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Pure 41 Binomial (Factorial Notation)

1. $5 \times 4 \times 3 \times 2 \times 1 = 120$

2. $\binom{5}{3} = \dfrac{5!}{3! \times 2!}$

   $= \dfrac{5 \times 4 \times \cancel{3!}}{\cancel{3!} \times 2!}$

   $= \dfrac{20}{2}$

   $= 10$

3. $1, 4, 6, 4, 1$

   $\therefore r = 1$ or $r = 3$

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1. $m = 15$ or $m = 3$

2. $n!$

3. $n = 16 - 5$

   $n = 11$

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1. $\dfrac{n!}{1!(n-1)!}$

   $= \dfrac{n!}{1 \times (n-1)!}$

   $= \dfrac{n\cancel{(n-1)!}}{1 \times \cancel{(n-1)!}}$

   n

2. $\dfrac{n!}{3!(n-3)!}$

   $= \dfrac{n(n-1)(n-2)\cancel{(n-3)!}}{6\cancel{(n-3)!}}$

   $= \dfrac{n(n-1)(n-2)}{6}$

   $= \dfrac{n(n^2 - 3n + 2)}{6}$

   $= \dfrac{n^3 - 3n^2 + 2n}{6}$

42 The Binomial Expansion

1 (a)

$${}^4C_0(2)^4(3x)^0 + {}^4C_1(2)^3(3x)^1 + {}^4C_2(2)^2(3x)^2 + {}^4C_3(2)^1(3x)^3 + {}^4C_4(2)^0(3x)^4$$

$$1(16)(1) + 4(8)(3x) + 6(4)(9x^2) + 4(2)(27x^3) + 1(1)(81x^4)$$

$$16 + 96x + 216x^2 + 216x^3 + 81x^4$$

2

$${}^8C_0(1)^8\left(\frac{x}{4}\right)^0 + {}^8C_1(1)^7\left(\frac{x}{4}\right)^1 + {}^8C_2(1)^6\left(\frac{x}{4}\right)^2 + {}^8C_3(1)^5\left(\frac{x}{4}\right)^3$$

$$1(1)(1) + 8(1)\left(\frac{x}{4}\right) + 28(1)\left(\frac{x^2}{16}\right) + 56(1)\left(\frac{x^3}{64}\right)$$

$$1 + 2x + \frac{7}{4}x^2 + \frac{7}{8}x^3$$

3

$${}^5C_0(1)^5(-x)^0 + {}^5C_1(1)^4(-x)^1 + {}^5C_2(1)^3(-x)^2 + {}^5C_3(1)^2(-x)^3 + {}^5C_4(1)^1(-x)^4 + {}^5C_5(1)(-x)^5$$

$$1(1)(1) + 5(1)(-x) + 10(1)(x^2) + 10(1)(-x^3) + 5(1)(x^4) + 1(1)(-x)^5$$

$$1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$$

1 (a)

$${}^5C_0(a)^5(b)^0 + {}^5C_1(a)^4(b)^1 + {}^5C_2(a)^3(b)^2 + {}^5C_3(a)^2(b)^3 + {}^5C_4(a)^1(b)^4 + {}^5C_5(a)^0(b)^5$$

$$1(a^5)(1) + 5(a^4)(b^1) + 10(a^3)(b^2) + 10(a^2)(b^3) + 5(a^1)(b^4) + 1(1)(b^5)$$

$$a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

(b) $a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$

2

$${}^4C_0(2)^4\left(\frac{x}{2}\right)^0 + {}^4C_1(2)^3\left(\frac{x}{2}\right)^1 + {}^4C_2(2)^2\left(\frac{x}{2}\right)^2 + {}^4C_3(2)^1\left(\frac{x}{2}\right)^3 + {}^4C_4(2)^0\left(\frac{x}{2}\right)^4$$

$$1(16)(1) + 4(8)\left(\frac{x}{2}\right) + 6(4)\left(\frac{x^2}{4}\right) + 4(2)\left(\frac{x^3}{8}\right) + 1(1)\left(\frac{x^4}{16}\right)$$

$$16 + 16x + 6x^2 + x^3 + \frac{1}{16}x^4$$

3

$${}^{11}C_7(5)^4\left(-\frac{x}{3}\right)^7 = 330 \times 625 \times -\frac{x^7}{2187}$$

$$= -\frac{206250}{2187} \begin{matrix}\div 3\\ \div 3\end{matrix}$$

$$= -\frac{68750}{729}$$

1

$$1(a)^4\left(\frac{1}{a}\right)^0 + 4(a)^3\left(\frac{1}{a}\right)^1 + 6(a)^2\left(\frac{1}{a}\right)^2 + 4(a)^1\left(\frac{1}{a}\right)^3 + 1(a)^0\left(\frac{1}{a}\right)^4$$

$$a^4 + 4a^2 + 6 + \frac{4}{a^2} + \frac{1}{a^4}$$

$$\therefore \left(a - \frac{1}{a}\right)^4 = a^4 - 4a^2 + 6 - \frac{4}{a^2} + \frac{1}{a^4}$$

42 Continued

1 continued

$\therefore\ a^4 + 4a^2 + 6 + \frac{4}{a^2} + \frac{1}{a^4}$

$+\ a^4 - 4a^2 + 6 - \frac{4}{a^2} + \frac{1}{a^4}$

$2a^4 + 12 + \frac{2}{a^4}$

Common factor $\frac{2}{a^4}$

$\frac{2}{a^4}\left(a^8 + 6a^4 + 1\right)$

2 (a) $n+1$

(b) ${}^{n}C_{n-6}\,(a)^{n-6}(b)^{6}$

or $\binom{n}{n-6}\left(a^{n-6}\right)\left(b^{6}\right)$

3 Yes there will be an odd number of terms in the expansion.

The middle term will be $x^{\frac{n}{2}} \times \frac{1}{x^{\frac{n}{2}}}$ which will give a constant or a term independent of x.

The coeffient will be $\binom{n}{\frac{n}{2}}$ which is not relevant

43 Solving Binomial Problems (Yr1 Pure)

1 a

$^4C_0(1)^4(2ax)^0 + {}^4C_1(1)^3(2ax)^1 + {}^4C_2(1)^2(2ax)^2 + {}^4C_3(1)^1(2ax)^3 + {}^4C_4(1)^0(2ax)^4$

$1(1)(1) + 4(1)(2ax) + 6(1)(4a^2x^2) + 4(1)(8a^3x^3) + 1(1)(16a^4x^4)$

$1 + 8ax + 24a^2x^2 + 32a^3x^3 + 16a^4x^4$

b $8ax = 24x$

$8a = 24$

$a = 3$

c $24(3)^2 = 216$

2 a $^7C_0(1)^7(x)^0 + {}^7C_1(1)^6(x)^1 + {}^7C_2(1)^5(x)^2$

$1(1)(1) + 7(1)(x) + 21(1)(x^2)$

$1 + 7x + 21x^2$

b $(1-x)(1+7x+21x^2)$

$1 + 7x + 21x^2$
$- x - 7x^2$

$1 + 6x + 14x^2$

3 $^6C_1(2)^5(px)^1 = 960x$

$6(32)(px) = 960x$

$192p = 960$

$p = \frac{960}{192}$

$p = 5$

1 a $^6C_0(p)^6(3x)^0 + {}^6C_1(p)^5(3x)^1 + {}^6C_2(p)^4(3x)^2$

$1(p^6)(1) + 6(p^5)(3x) + 15(p^4)(9x^2)$

$p^6 + 18p^5x + 135p^4x^2$

b $2(135p^4) = 18p^5$

$270p^4 = 18p^5$

$18p^5 - 270p^4 = 0$

$18p^4(p-15) = 0$

$p^4(p-15) = 0$

c $p = 15$

d $18 \times 15^5 = 13668750$

2 a

$1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$

b $1 + 5(-2y) + 10(-2y)^2 + [10(-2y)^3 + 5(-2y)^4 + (-2y)^5]$

$1 - 10y + 40y^2$

1 a

$(3+x)\left[^7C_0(1)^7(px)^0 + {}^7C_1(1)^6(px)^1 + {}^7C_2(1)^5(px)^2 + \ldots\right]$

$(3+x)\left[1 + 7px + 21p^2x^2 + \ldots 35p^3x^3\right)$

$3 + 21px + 63p^2x^2$
$+ x + 7px^2$

$3 + (21p+1)x + (63p^2+7p)x^2 + \ldots$

b $63p^2 + 7p = 238$

$9p^2 + p - 34 = 0$

$p = \frac{17}{9}$ or $p = -2$ ✓

$\therefore p = -2$

Term in x^3

$(3+x)(\ldots 21p^2x^2 + 35p^3x^3)$

$105p^3 + 21p^2 \quad \therefore 105(-2)^3 + 21(-2)^2$

$= -756$

43 Continued

2 $(p-x)\left[1+8(2x)+28(4x^2)+\right]$

$(p-x)\left[1+16x+112x^2+\cdots\right]$

$p + 16px + 112px^2$
$\quad - x \quad - 16x^2 + \cdots$

$p + \underbrace{(16p-1)}_{\text{Must}=0}x + (112p-16)x^2$

$\therefore 16p - 1 = 0$
$p = \frac{1}{16}$

First term: $\frac{1}{16}$
Second term $112\left(\frac{1}{16}\right) - 16 = -9$

$\therefore A = \frac{1}{16}, B = -9$

3 Highest power of x will be

$x(x^3)^n \quad \therefore x^{1+3n} = x^{19}$

$3n+1 = 19, n = 6$

Questions at www.m4ths.com
Steve Blades

44 Binomial Estimations

1 (a) $\binom{7}{0}(1)^7(x)^0 + \binom{7}{1}(1)^6(x)^1 + \binom{7}{2}(1)^3(x)^2$

$1(1)(1) + 7(1)(x) + 21(1)(x^2)$

$1 + 7x + 21x^2$

(b) let $x = 0.01$

$1 + 7(0.01) + 21(0.01)^2$

$1 + 0.07 + 0.0021 = \underline{1.0721}$

2 (a)

$\binom{12}{0}(1)^{12}(-2x)^0 + \binom{12}{1}(1)^{11}(-2x)^1 + \binom{12}{2}(1)^{10}(-2x)^2 + \binom{12}{3}(1)^9(-2x)^3$

$1(1)(1) + 12(1)(-2x) + 66(1)(4x^2) + 220(1)(-8x^3)$

$1 - 24x + 264x^2 - 1760x^3$

(b) $1 - 2x = 0.96$

$1 - 0.96 = 2x$

$0.04 = 2x$

$0.02 = x$

$1 - 24(0.02) + 264(0.02)^2 - 1760(0.02)^3$

$= \frac{1911}{3125} = 0.61152$

1 (a) $\binom{8}{0}(2)^8\left(-\frac{x}{4}\right)^0 + \binom{8}{1}(2)^7\left(-\frac{x}{4}\right)^1 + \binom{8}{2}(2)^6\left(-\frac{x}{4}\right)^2$

$1(256)(1) + 8(128)\left(-\frac{x}{4}\right) + 28(64)\left(\frac{x^2}{16}\right)$

$256 - 256x + 112x^2 + \ldots$

(b) $2 - \frac{x}{4} = 1.99$

$0.01 = \frac{x}{4}$

$0.04 = x$

$\therefore 256 - 256(0.04) + 112(0.04)^2$

$= 245.9392$

(c) $\frac{1.99^8 - 245.9392}{1.99^8} \times 100$

$= -7.24 \times 10^{-4}\,\%$ ✓

2 (a) $\binom{9}{0}(5)^9\left(-\frac{x}{3}\right)^0 + \binom{9}{1}(5)^8\left(-\frac{x}{3}\right)^1 + \binom{9}{2}(5)^7\left(-\frac{x}{3}\right)^2$

$1(1953125) + 9\left(390625\left(-\frac{x}{3}\right)\right)$

$+ 36(78125)\left(\frac{x^2}{9}\right)$

$1953125 - 1171875x + 312500x^2$

(b) $\left(\frac{1}{5} + x\right)(1953125 - 1171875x)$

$\frac{1}{5}(1953125) + x\left(1953125 - \frac{1171875}{5}\right)$

$= 390625 + 1718750x$ ✓

1 $\binom{n}{0}(a)^n(x)^0 + \binom{n}{1}(a)^{n-1}(x)^1 + \binom{n}{2}(a)^{n-2}(x)^2 + \ldots$

$(1)(a)(1) + n(a)^{n-1}(x) + \frac{n(n-1)}{2}(a^{n-2})(x^2)$

$a + n(a)^{n-1}(x) + \frac{n(n-1)}{2}a^{n-2}(x^2)$

The expansion of $(a-x)^n$ will be

$a - n(a)^{n-1}(x) + \frac{n(n-1)}{2}(a)^{n-2}(x^2)$

$\therefore (a + na^{n-1}x + \ldots)(a - na^{n-1}x + \ldots)$

$a^2 + \cancel{na^n}x - \cancel{na^n}x \ldots\ldots = \underline{\underline{a^2}}$

2 $1(5)^8(1) + 8(5)^7(-4x) + 28(5)^6(16x^2) + 56(5)^5(-64x^3)$

$390625 - 2500000x + 7000000x^2$

$- 11200000x^3 \ldots$

let $x = 0.02$ and sub in

$390625 - 2500000(0.02) + 7000000(0.02)^2$

$- 11200000(0.02)^3 \ldots = \underline{343335.4}$

Pure Yr1 45 The Cosine Rule.

1 (a) If a right angled triangle

$a^2+b^2=c^2$

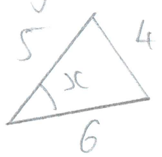
$4^2+5^2 = 16+25$

$= 41$

$\sqrt{41} = 6.403\ldots$

∴ not a right angled triangle as $\sqrt{41} \neq 6$.

(b)



$\cos x = \frac{5^2+6^2-4^2}{2(5)(6)}$

$x = \cos^{-1}\left(\frac{3}{4}\right)$

$x = 41.4°$

2 $7+11+\sqrt{7^2+11^2-2(7)(11)\cos 80}$

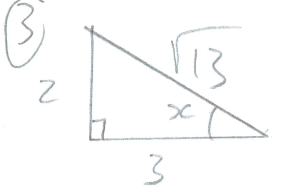
$= 18+\sqrt{143.2\ldots}$

$= 18+11.96\ldots$

$= 29.969\ldots$

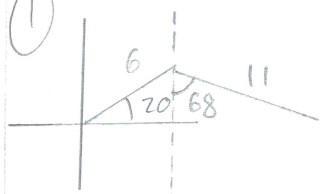
∴ 30.0 to 3 S.F.

3

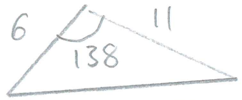


$\cos x = \frac{3}{\sqrt{13}}$

1



(a)

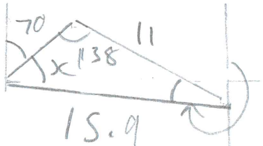


$\sqrt{6^2+11^2-2(6)(11)\cos 138}$

$= 15.971\ldots$

∴ 16.0 km

(b)



$\frac{\sin x}{11} = \frac{\sin 138}{15.9\ldots}$

$x = \sin^{-1}\left(\frac{11\sin 138}{15.9\ldots}\right)$

$x = 27.44$

∴ $180+70+27.44$

$= 277°$

2 $\cos y = \frac{7^2+3^2-x^2}{2(7)(3)}$

(a) $= \frac{49+9-x^2}{42}$

$= \frac{58-x^2}{42}$ ✓

(b) $\cos(30) = \frac{\sqrt{3}}{2}$

∴ $\frac{\sqrt{3}}{2} = \frac{58-x^2}{42}$

$\frac{42\sqrt{3}}{2} = 58-x^2$

$x^2 = 58-\frac{42\sqrt{3}}{2}$

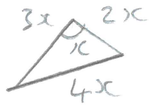
$x^2 = 58-21\sqrt{3}$

$x = \pm\sqrt{58-21\sqrt{3}}$

x is a length ∴

$x = \sqrt{58-21\sqrt{3}}$ ✓

3



$\cos x = \frac{(3x)^2+(2x)^2-(4x)^2}{2(3x)(2x)}$

$= \frac{9\cancel{x^2}+4\cancel{x^2}-16\cancel{x^2}}{12\cancel{x^2}}$

$= \frac{+13-16}{12}$

$= -\frac{3}{12}$

$= -\frac{1}{4}$ ✓

1 (a) $\cos y = \frac{x^2+(x+2)^2-(x+1)^2}{2(x)(x+2)}$

$= x^2+x^2+4x+4-[x^2+2x+1]$

$= \frac{x^2+2x+3}{2(x)(x+2)}$

(b) $\cos 90 = 0$ ∴ if w is a right angle,

$\frac{x^2+x+1}{2x+3} = 0$

∴ $x^2+x+1=0$

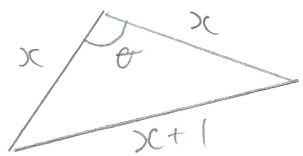
$a=1, b=1, c=1$

b^2-4ac

$1-4(1)(1) = -3$

∴ no real roots as $b^2-4ac<0$

2



let x be the shortest side.

For obtuse angles cos θ will be negative.
x will have to be >0 as it's a length.

45 Continued

$$\cos\theta = \frac{x^2 + x^2 - (x+1)^2}{2(x)(x)}$$

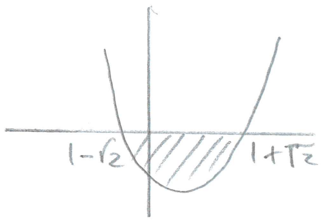
$$= \frac{2x^2 - (x^2 + 2x + 1)}{2x^2}$$

$$= \frac{x^2 - 2x - 1}{2x^2}$$

$\therefore x^2 - 2x - 1 < 0$

$(x-1)^2 - 1 - 1 < 0$

$(x-1)^2 - 2 < 0$



$1 - \sqrt{2} < x < 1 + \sqrt{2}$

x is a length $\therefore$

$0 < x < 1 + \sqrt{2}$

$\therefore x + 1 < 2 + \sqrt{2}$ ✓

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