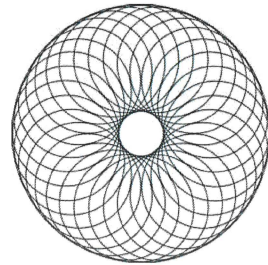


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|--|---|--|
| <ul style="list-style-type: none">(1) Indices(2) Expanding Brackets(3) Factorising Expressions(4) More Indices (Negative and Fractional)(5) Working with Surds(6) Solving Quadratic Equations(7) Completing the Square for Quadratics Expressions(8) Function Notation(9) Sketching Quadratic Graphs(10) The Discriminant for Quadratic Equations(11) Applications of Quadratics Equations(12) Solving Linear Simultaneous Equations(13) Linear & Non-Linear Simultaneous Equations(14) Graphing Simultaneous Equations(15) Linear Inequalities(16) Quadratic Inequalities(17) Graphing Inequalities(18) Shading Inequalities(19) Cubic Graphs(20) Quartic Graphs(21) Reciprocal Graphs(22) The Intersection of Graphs(23) Transforming Graphs (Translations)(24) Transforming Graphs (Stretching/Reflecting)(25) Straight Line Graphs in the form $y = mx + c$(26) More Straight Line Graphs(27) Straight Line Graphs (Parallel & Perpendicular)(28) The Geometry of Straight Lines(29) The Application of Linear Graphs(30) Circle Geometry Midpoint & Perpendicular | <ul style="list-style-type: none">(31) The Equation of a Circle(32) Circles and Straight Lines (Intersections)(33) Circles (Tangents and Chords)(34) Circles and Triangles(35) Algebraic Fractions(36) Polynomial Division(37) The Factor and Remainder Theorem(38) An Introduction to Mathematical Proof(39) Methods of Proof(40) Binomial Expansion (Using Pascal's Triangle)(41) Binomial Expansion (Factorial Notation)(42) Binomial Expansion (The $\binom{n}{r}$ Method)(43) Binomial Expansion (Problem Solving)(44) Binomial Expansion (Estimations and Approximations)(45) The Cosine Rule(46) The Sine Rule(47) Areas of a Triangles(48) Triangles (Problem Solving)(49) Sine, Cosine & Tangent Graphs(50) Transforming Graphs (Trigonometry)(51) The 'CAST' Diagram for Trig Ratios(52) Trigonometry (Exact Values)(53) Proving Trigonometric Identities(54) Solving Basic Trigonometric Equations(55) More Challenging Trigonometric Equations(56) Using Identities to Solve Trig Equations(57) Vectors (Introduction) | <ul style="list-style-type: none">(58) Vector Notation (Column and i and j form)(59) Vectors (Magnitude and Direction)(60) Vectors (Position and Direction Vectors)(61) Vector Geometry(62) Application of Vectors(63) Differentiation (Gradients of Curves)(64) Differentiation from 1st Principles(65) Differentiating x^n (Basic Powers of)(66) Differentiation (Quadratic Expression)(67) Differentiation (Multiple Terms)(68) Differentiation (Gradients, Tangents and Normals)(69) Differentiation (Increasing and Decreasing Functions)(70) Differentiation (Stationary Points)(71) Differentiation (Gradient Functions)(72) The Applications of Differentiation(73) Integration (Basic Expressions (x^n))(74) Indefinite Integrals(75) Integration (Finding c and Finding Functions)(76) Integration (Definite Integrals)(77) Integration (Basic Areas Under Curves)(78) Integration ('Negative and Positive Areas')(79) Integration (Areas between Curves and Lines)(80) Basic Exponential Functions(81) 'The' Exponential Function $y = e^x$(82) Applications of Basic Exponential Models(83) Logarithms (Simplifying & Evaluating)(84) Logarithms (The Log Laws)(85) Logarithms (Log and Exponential Equations) |
|--|---|--|

Pure **(41)** Binomial
(Factorial Notation)

(1) $5 \times 4 \times 3 \times 2 \times 1 = 120$

(2) $\binom{5}{3} = \frac{5!}{3! \times 2!}$
 $= \frac{5 \times 4 \times 3!}{3! \times 2!}$
 $= \frac{20}{2}$
 $= 10$

(3) 1, 4, 6, 4, 1
 $\therefore r=1$ or $r=3$

(1) $m=15$ or $m=3$

(2) $n!$

(3) $n=16-5$
 $n=11$

(1) $\frac{n!}{1! (n-1)!}$

$= \frac{n!}{1 \times (n-1)!}$

$= \frac{n(n-1)!}{1 \times (n-1)!}$
 n

(2) $\frac{n!}{3! (n-3)!}$

$= \frac{n(n-1)(n-2)(n-3)!}{6 (n-3)!}$

$= \frac{n(n-1)(n-2)}{6}$

$= \frac{n(n^2 - 3n + 2)}{6}$

$= \frac{n^3 - 3n^2 + 2n}{6}$

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42 The Binomial Expansion

1a

$$\binom{4}{0} 7^4 (3x)^0 + \binom{4}{1} 7^3 (3x)^1 + \binom{4}{2} 7^2 (3x)^2 + \binom{4}{3} 7^1 (3x)^3 + \binom{4}{4} 7^0 (3x)^4$$

$$1(16)(1) + 4(8)(3x) + 6(4)(9x^2) + 4(7)(27x^3) + 1(1)(81x^4)$$

$$16 + 96x + 216x^2 + 216x^3 + 81x^4$$

2

$$\binom{8}{0} \left(\frac{x}{4}\right)^0 + \binom{8}{1} \left(\frac{x}{4}\right)^1 + \binom{8}{2} \left(\frac{x}{4}\right)^2 + \binom{8}{3} \left(\frac{x}{4}\right)^3$$

$$1(1)(1) + 8(1)\left(\frac{x}{4}\right) + 28(1)\left(\frac{x^2}{16}\right) + 56(1)\left(\frac{x^3}{64}\right)$$

$$1 + 2x + \frac{7}{4}x^2 + \frac{7}{8}x^3$$

3

$$\binom{5}{0} (1)^5 (-x)^0 + \binom{5}{1} (1)^4 (-x)^1 + \binom{5}{2} (1)^3 (-x)^2 + \binom{5}{3} (1)^2 (-x)^3 + \binom{5}{4} (1)^1 (-x)^4 + \binom{5}{5} (1)^0 (-x)^5$$

$$1(1)(1) + 5(1)(-x) + 10(1)(x^2) + 10(1)(-x^3) + 5(1)(x^4) + 1(1)(-x^5)$$

$$1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$$

1a

$$\binom{5}{0} (a)^5 (b)^0 + \binom{5}{1} (a)^4 (b)^1 + \binom{5}{2} (a)^3 (b)^2 + \binom{5}{3} (a)^2 (b)^3 + \binom{5}{4} (a)^1 (b)^4 + \binom{5}{5} (a)^0 (b)^5$$

$$1(a^5)(1) + 5(a^4)(b^1) + 10(a^3)(b^2) + 10(a^2)(b^3) + 5(a^1)(b^4) + 1(1)(b^5)$$

$$a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

6

$$a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$$

7

$$\binom{4}{0} 2^4 \left(\frac{x}{2}\right)^0 + \binom{4}{1} 2^3 \left(\frac{x}{2}\right)^1 + \binom{4}{2} 2^2 \left(\frac{x}{2}\right)^2 + \binom{4}{3} 2^1 \left(\frac{x}{2}\right)^3 + \binom{4}{4} 2^0 \left(\frac{x}{2}\right)^4$$

$$1(16)(1) + 4(8)\left(\frac{x}{2}\right) + 6(4)\left(\frac{x^2}{4}\right) + 4(2)\left(\frac{x^3}{8}\right) + 1(1)\left(\frac{x^4}{16}\right)$$

$$16 + 16x + 6x^2 + x^3 + \frac{1}{16}x^4$$

3

$$\binom{11}{7} (5)^4 \left(-\frac{x}{3}\right)^7 = 330 \times 625 \times \frac{-x^7}{2187}$$

$$= \frac{-206250}{2187} \div 3$$

$$= \frac{-68750}{729}$$

1

$$1(a)^4 \left(\frac{1}{a}\right)^0 + 4(a)^3 \left(\frac{1}{a}\right)^1 + 6(a)^2 \left(\frac{1}{a}\right)^2 + 4(a)^1 \left(\frac{1}{a}\right)^3 + 1(a)^0 \left(\frac{1}{a}\right)^4$$

$$a^4 + 4a^2 + 6 + \frac{4}{a^2} + \frac{1}{a^4}$$

$$\therefore \left(a - \frac{1}{a}\right)^4 = a^4 - 4a^2 + 6 - \frac{4}{a^2} + \frac{1}{a^4}$$

(42) Continued

① continued

$$\therefore a^4 + 4a^2 + 6 + \frac{4}{a^2} + \frac{1}{a^4} \\ + a^4 - 4a^2 + 6 - \frac{4}{a^2} + \frac{1}{a^4}$$

$$2a^4 + 12 + \frac{2}{a^4}$$

Common factor $\frac{2}{a^4}$

$$\frac{2}{a^4} (a^8 + 6a^4 + 1)$$

② (a) $n+1$

$$(b) \binom{n}{n-6} \binom{n-6}{6} a^6 b^6$$

$$\text{or } \binom{n}{n-6} \binom{n-6}{6} (a^6 b^6)$$

③ Yes there will be an odd number of terms in the expansion.

The middle term will be

$$x^{\frac{n}{2}} \times \frac{1}{x^{\frac{n}{2}}}$$

give a constant or a term independent of x .

The coefficient will be

$$\binom{n}{\frac{n}{2}}$$

which is not relevant

43 Solving Binomial Problems (Y1 Pure)

1a

$$\binom{4}{0} (1)^4 (2ax)^0 + \binom{4}{1} (1)^3 (2ax)^1 + \binom{4}{2} (1)^2 (2ax)^2 + \binom{4}{3} (1)^1 (2ax)^3 + \binom{4}{4} (1)^0 (2ax)^4$$

$$1(1)(1) + 4(1)(2ax) + 6(1)(4a^2x^2) + 4(1)(8a^3x^3) + 1(1)(16a^4x^4)$$

$$1 + 8ax + 24a^2x^2 + 32a^3x^3 + 16a^4x^4$$

b) $8ax = 24x$
 $8a = 24$
 $a = 3$

c) $24(3)^2 = 216$

2a) $\binom{7}{0} (1)^7 (x)^0 + \binom{7}{1} (1)^6 (x)^1 + \binom{7}{2} (1)^5 (x)^2$
 $1(1)(1) + 7(1)(x) + 21(1)(x^2)$
 $1 + 7x + 21x^2$

b) $(1-x)(1+7x+21x^2)$
 $1 + 7x + 21x^2$
 $-x - 7x^2$

 $1 + 6x + 14x^2$

3) $\binom{6}{0} (2)^5 (px)^1 = 960x$
 $6(32)(px) = 960x$
 $192p = 960$
 $p = \frac{960}{192}$
 $p = 5$

1a) $\binom{6}{0} (p)^6 (3x)^0 + \binom{6}{1} (p)^5 (3x)^1 + \binom{6}{2} (p)^4 (3x)^2$
 $1(p^6)(1) + 6(p^5)(3x) + 15(p^4)(9x^2)$
 $p^6 + 18p^5x + 135p^4x^2$

b) $2(135p^4) = 18p^5$
 $270p^4 = 18p^5$
 $18p^5 - 270p^4 = 0$
 $18p^4(p-15) = 0$
 $p^4(p-15) = 0$

c) $p = 15$

d) $18 \times 15^5 = 13668750$

2a) $1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$
 b) $1 + 5(-2y) + 10(-2y)^2 + [10(-2y)^3 + 5(-2y)^4 + (-2y)^5]$
 $1 - 10y + 40y^2$

1a) $(3+x) \left[\binom{7}{0} (1)^7 (px)^0 + \binom{7}{1} (1)^6 (px)^1 + \binom{7}{2} (1)^5 (px)^2 + \dots \right]$
 $(3+x) [1 + 7px + 21p^2x^2 + \dots + 35p^3x^3]$
 $3 + 21px + 63p^2x^2 + \dots$
 $+ x + 7px^2 + \dots$
 $3 + (21p+1)x + (63p^2+7p)x^2 + \dots$

b) $63p^2 + 7p = 238$
 $9p^2 + p - 34 = 0$
 $p = \frac{17}{9}$ or $p = -2$ ✓
 $\therefore p = -2$

Term in x^3
 $(3+x)(+21p^3x^2 + 35p^3x^3)$
 $105p^3 + 21p^2 \therefore 105(-2)^3 + 21(-2)^2$
 $= -756$

43) Continued

$$(p-x) [1 + 8(2x) + 28(4x^2) + \dots]$$

$$(p-x) [1 + 16x + 112x^2 + \dots]$$

$$p + 16px + 112px^2 - x - 16x^2 + \dots$$

$$p + (16p-1)x + (112p-16)x^2$$

Must = 0

$$\therefore 16p-1=0$$

$$p = \frac{1}{16}$$

First term: $\frac{1}{16}$

$$\text{Second term } 112\left(\frac{1}{16}\right) - 16 = -9$$

$$\therefore A = \frac{1}{16}, B = -9$$

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(3) Highest power of x
will be

$$x(x^3)^n \therefore x^{1+3n} = x^{19}$$

$$3n+1=19, n=6 //$$

44 Binomial Estimators

1(a)
$$\sum_{r=0}^7 \binom{7}{r} (1)^{7-r} (x)^r = \binom{7}{0} (1)^7 (x)^0 + \binom{7}{1} (1)^6 (x)^1 + \binom{7}{2} (1)^5 (x)^2 + \dots + \binom{7}{7} (1)^0 (x)^7$$

$$= (1)(1) + 7(1)(x) + 21(1)(x^2) + \dots + 7(1)(x^6) + 1(x^7)$$

$$1 + 7x + 21x^2 + \dots + 7x^6 + x^7$$

(b) let $x = 0.01$

$$1 + 7(0.01) + 21(0.01)^2 + \dots + 7(0.01)^6 + (0.01)^7$$

$$1 + 0.07 + 0.0021 + \dots + 0.000007 + 0.00000001 = \underline{1.0721}$$

2(a)
$$\sum_{r=0}^{12} \binom{12}{r} (1)^{12-r} (-2x)^r = \binom{12}{0} (1)^{12} (-2x)^0 + \binom{12}{1} (1)^{11} (-2x)^1 + \binom{12}{2} (1)^{10} (-2x)^2 + \dots + \binom{12}{12} (1)^0 (-2x)^{12}$$

$$= (1)(1) + 12(1)(-2x) + 66(1)(4x^2) + 220(1)(-8x^3) + \dots + 12(1)(-2x)^{11} + 1(-2x)^{12}$$

$$1 - 24x + 264x^2 - 1760x^3 + \dots - 220(1)(-8x^3) + 12(1)(-2x)^{11} + 1(-2x)^{12}$$

(b) $1 - 2x = 0.96$
 $1 - 0.96 = 2x$
 $0.04 = 2x$
 $0.02 = x$

$$1 - 24(0.02) + 264(0.02)^2 - 1760(0.02)^3 + \dots - 220(1)(-8x^3) + 12(1)(-2x)^{11} + 1(-2x)^{12}$$

$$= \frac{1911}{3125} = 0.61152$$

1(a)
$$\sum_{r=0}^8 \binom{8}{r} \left(\frac{-x}{4}\right)^r = \binom{8}{0} \left(\frac{-x}{4}\right)^0 + \binom{8}{1} \left(\frac{-x}{4}\right)^1 + \binom{8}{2} \left(\frac{-x}{4}\right)^2 + \dots + \binom{8}{8} \left(\frac{-x}{4}\right)^8$$

$$= 1 \binom{8}{0} (1) + 8 \binom{8}{1} \left(\frac{-x}{4}\right) + 28 \binom{8}{2} \left(\frac{x^2}{16}\right) + \dots + 8 \binom{8}{8} \left(\frac{-x}{4}\right)^8$$

$$= 256 - 256x + 112x^2 + \dots - 256x^8 + x^8$$

(b) $2 \cdot \frac{x}{4} = 1.99$
 $0.01 = \frac{x}{4}$
 $0.04 = x$

$$\therefore 256 - 256(0.04) + 112(0.04)^2 + \dots - 256(0.04)^8 + (0.04)^8$$

$$= 245.9392$$

(c)
$$\frac{1.99^8 - 245.9392}{1.99^8} \times 100$$

$$= -7.24 \times 10^{-4} \% \quad \checkmark$$

2(a)
$$\sum_{r=0}^9 \binom{9}{r} \left(\frac{-x}{3}\right)^r = \binom{9}{0} \left(\frac{-x}{3}\right)^0 + \binom{9}{1} \left(\frac{-x}{3}\right)^1 + \binom{9}{2} \left(\frac{-x}{3}\right)^2 + \dots + \binom{9}{9} \left(\frac{-x}{3}\right)^9$$

$$= 1 \binom{9}{0} (1) + 9 \binom{9}{1} \left(\frac{-x}{3}\right) + 36 \binom{9}{2} \left(\frac{x^2}{9}\right) + \dots + 9 \binom{9}{9} \left(\frac{-x}{3}\right)^9$$

$$= 1953125 - 1171875x + 312500x^2 + \dots - 1171875x^8 + x^9$$

(b)
$$\left(\frac{1}{5} + x\right) \left(1953125 - 1171875x\right)$$

$$= \frac{1}{5} (1953125) + x \left(1953125 - \frac{1171875}{5}\right) - 1171875x^2 - \dots$$

$$= 390625 + 1718750x \quad \checkmark$$

1
$$\sum_{r=0}^n \binom{n}{r} (a)^{n-r} (x)^r = \binom{n}{0} (a)^n (x)^0 + \binom{n}{1} (a)^{n-1} (x)^1 + \binom{n}{2} (a)^{n-2} (x)^2 + \dots + \binom{n}{n} (a)^0 (x)^n$$

$$= (1)(a)(1) + n(a)^{n-1}(x) + \frac{n(n-1)}{2}(a)^{n-2}(x^2) + \dots + a + n(a)^{n-1}(x) + \frac{n(n-1)}{2}a^{n-2}(x^2) + \dots$$

The expansion of $(a-x)^n$ will be

$$a - n(a)^{n-1}(x) + \frac{n(n-1)}{2}(a)^{n-2}(x^2) - \dots + (-1)^n (a)^n$$

$\therefore (a + na^{n-1}x + \dots)(a - na^{n-1}x + \dots)$

$$a^2 + \cancel{na^n x} - \cancel{na^n x} - \dots = \underline{\underline{a^2}}$$

2
$$1 \binom{5}{0} (1) + 5 \binom{5}{1} (-4x) + 10 \binom{5}{2} (16x^2) + 10 \binom{5}{3} (-64x^3) + 5 \binom{5}{4} (256x^4) + 1 \binom{5}{5} (-1024x^5)$$

$$= 390625 - 2500000x + 7000000x^2 - 11200000x^3 + \dots - 1024x^5$$

let $x = 0.02$ and sub in

$$390625 - 2500000(0.02) + 7000000(0.02)^2 - 11200000(0.02)^3 + \dots - 1024(0.02)^5$$

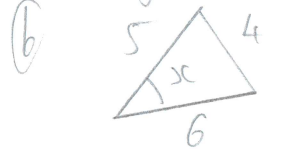
$$= 343335.4$$

Pure X1 (45) The Cosine Rule.

1) If a right angled triangle $a^2 + b^2 = c^2$

$$4^2 + 5^2 = 16 + 25 = 41$$

$\sqrt{41} = 6.403 \dots$
 \therefore not a right angled triangle as $\sqrt{41} \neq 6$.

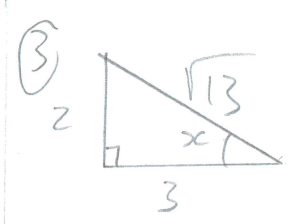


$$\cos x = \frac{5^2 + 6^2 - 4^2}{2(5)(6)}$$

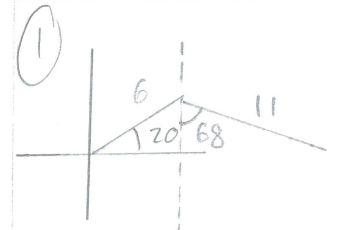
$$x = \cos^{-1}\left(\frac{3}{4}\right)$$

$$x = 41.4^\circ$$

$$\begin{aligned} 2) & 7 + 11 + \sqrt{7^2 + 11^2 - 2(7)(11)\cos 80} \\ &= 18 + \sqrt{143 \cdot 2} \\ &= 18 + 11.96 \dots \\ &= 29.969 \dots \\ &\therefore 30.0 \text{ to } 3\text{s.f.} \end{aligned}$$

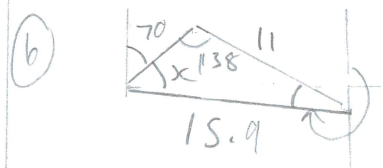


$$\cos x = \frac{3}{\sqrt{13}}$$



$$\begin{aligned} a) & \sqrt{6^2 + 11^2 - 2(6)(11)\cos 138} \\ &= 15.971 \dots \end{aligned}$$

$\therefore 16.0 \text{ km}$



$$\begin{aligned} \frac{\sin x}{11} &= \frac{\sin 138}{15.9} \\ x &= \sin^{-1}\left(\frac{11 \sin 138}{15.9}\right) \\ x &= 27.44 \\ \therefore & 180 + 70 + 27.44 \\ &= 277^\circ \end{aligned}$$

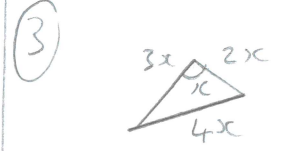
$$2) \cos y = \frac{7^2 + 3^2 - x^2}{2(7)(3)}$$

$$\begin{aligned} a) & \frac{49 + 9 - x^2}{42} \\ &= \frac{58 - x^2}{42} \checkmark \end{aligned}$$

$$\begin{aligned} b) \cos(30) &= \frac{\sqrt{3}}{2} \\ \therefore \frac{\sqrt{3}}{2} &= \frac{58 - x^2}{42} \end{aligned}$$

$$\begin{aligned} \frac{42\sqrt{3}}{2} &= 58 - x^2 \\ x^2 &= 58 - \frac{42\sqrt{3}}{2} \\ x^2 &= 58 - 21\sqrt{3} \end{aligned}$$

$$\begin{aligned} x &= \pm \sqrt{58 - 21\sqrt{3}} \\ x \text{ is a length } \therefore & \\ x &= \sqrt{58 - 21\sqrt{3}} \checkmark \end{aligned}$$



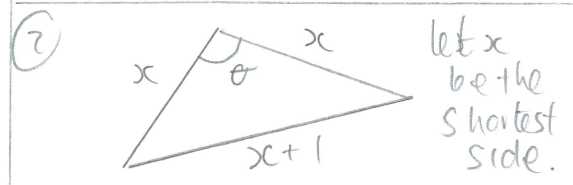
$$\begin{aligned} \cos x &= \frac{(3x)^2 + (2x)^2 - (4x)^2}{2(3x)(2x)} \\ &= \frac{9x^2 + 4x^2 - 16x^2}{12x^2} \end{aligned}$$

$$\begin{aligned} &= \frac{+13 - 16}{12} \\ &= \frac{-3}{12} \\ &= -\frac{1}{4} \checkmark \end{aligned}$$

$$\begin{aligned} 1) a) \cos y &= \frac{x^2 + (x+2)^2 - (x+1)^2}{2(x)(x+2)} \\ &= \frac{x^2 + x^2 + 4x + 4 - [x^2 + 2x + 1]}{2(x)(x+2)} \\ &= \frac{x^2 + 2x + 3}{2(x)(x+2)} \end{aligned}$$

$$\begin{aligned} b) \cos 90 &= 0 \therefore \text{if } w \text{ is a right angle,} \\ \frac{x^2 + x + 1}{2x + 3} &= 0 \end{aligned}$$

$$\begin{aligned} \therefore x^2 + x + 1 &= 0 \\ a=1, b=1, c=1 \\ b^2 - 4ac &= 1 - 4(1)(1) = -3 \\ \therefore \text{no real roots as } b^2 - 4ac < 0 \end{aligned}$$



For obtuse angles $\cos \theta$ will be negative. x will have to be > 0 as it's a length.

(45) Continued

$$\cos \theta = \frac{x^2 + x^2 - (x+1)^2}{2(x)(x)}$$

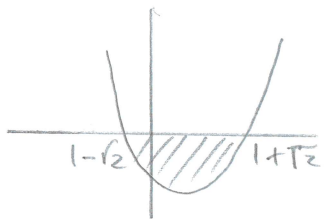
$$= \frac{2x^2 - (x^2 + 2x + 1)}{2x^2}$$

$$= \frac{x^2 - 2x - 1}{2x^2}$$

$$\therefore x^2 - 2x - 1 < 0$$

$$(x-1)^2 - 1 - 1 < 0$$

$$(x-1)^2 - 2 < 0$$



$$1 - \sqrt{2} < x < 1 + \sqrt{2}$$

x is a length \therefore

$$0 < x < 1 + \sqrt{2}$$

$$\therefore x+1 < 2 + \sqrt{2} \checkmark$$

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