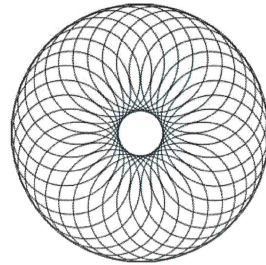


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Pure (36) Dividing Polynomials

$$\textcircled{1} \quad x+1 \overline{) \begin{array}{r} x^2 + 3x - 18 \\ x^3 + 4x^2 - 15x - 18 \\ \underline{x^3 + x^2} \\ 0 + 3x^2 - 15x \\ \quad + 3x^2 + 3x \\ \quad \underline{ + 3x^2 + 3x} \\ \quad 0 - 18x - 18 \\ \quad \quad - 18x - 18 \\ \quad \quad \underline{ - 18x - 18} \\ \quad \quad \quad 0 + 0 \end{array}}$$

No remainder. $\rightarrow 0 + 0$

$$\textcircled{2} \quad x+2 \overline{) \begin{array}{r} x^2 + 11x + 30 \\ x^3 + 13x^2 + 52x + 60 \\ \underline{x^3 + 2x^2} \\ 0 + 11x^2 + 52x \\ \quad + 11x^2 + 22x \\ \quad \underline{ + 11x^2 + 22x} \\ \quad + 0 + 30x + 60 \\ \quad \quad + 30x + 60 \\ \quad \quad \underline{ + 30x + 60} \\ \quad \quad \quad 0 + 0 \end{array}}$$

$$\textcircled{a} \quad \begin{array}{r} 0 + 11x^2 + 52x \\ + 11x^2 + 22x \\ \hline + 0 + 30x + 60 \\ + 30x + 60 \\ \hline 0 + 0 \end{array}$$

$$\textcircled{b} \quad x^3 + 13x^2 + 52x + 60 \equiv (x+2)(x^2 + 11x + 30) \\ (x+2)(x+6)(x+5)$$

$$\textcircled{3} \quad x-3 \overline{) \begin{array}{r} x^2 + 6x + 2 \\ x^3 + 3x^2 - 16x + 7 \\ \underline{x^3 - 3x^2} \\ 0 + 6x^2 - 16x \\ \quad + 6x^2 - 18x \\ \quad \underline{ + 6x^2 - 18x} \\ \quad 0 + 20x + 7 \\ \quad \quad + 20x - 6 \\ \quad \quad \underline{ + 20x - 6} \\ \quad \quad \quad 0 + 13 \end{array}}$$

$$\underline{r=13}$$

$$\textcircled{1} \quad x+1 \overline{) \begin{array}{r} x^2 - x - 6 \\ x^3 + 0x^2 - 7x - 6 \\ \underline{x^3 + x^2} \\ 0 - x^2 - 7x \\ \quad - x^2 - x \\ \quad \underline{ - x^2 - x} \\ \quad 0 - 6x - 6 \\ \quad \quad - 6x - 6 \\ \quad \quad \underline{ - 6x - 6} \\ \quad \quad \quad 0 + 0 \end{array}}$$

$$\textcircled{a} \quad \begin{array}{r} 0 - x^2 - 7x \\ - x^2 - x \\ \hline 0 - 6x - 6 \\ - 6x - 6 \\ \hline 0 + 0 \end{array}$$

$$\textcircled{b} \quad (x+1)(x^2 - x - 6) \\ (x+1)(x-3)(x+2)$$

$\textcircled{2}$

$$\textcircled{2} \textcircled{a} \quad 2x+1 \overline{) \begin{array}{r} x^2 + 6x - 7 \\ 2x^3 + 13x^2 - 8x - 7 \\ \underline{2x^3 + x^2} \\ 0 + 12x^2 - 8x \\ \quad + 12x^2 + 6x \\ \quad \underline{ + 12x^2 + 6x} \\ \quad 0 - 14x - 7 \\ \quad \quad - 14x - 7 \\ \quad \quad \underline{ - 14x - 7} \\ \quad \quad \quad 0 + 0 \end{array}}$$

$$\textcircled{b} \quad g(x) \equiv (2x+1)(x^2 + 6x - 7) \\ 0 = (2x+1)(x+7)(x-1)$$

$$\therefore x = -\frac{1}{2}, x = -7, x = 1$$

$$\textcircled{3} \quad x-5 \overline{) \begin{array}{r} x^2 + x + 3 \\ x^3 - 4x^2 - 2x - 15 \\ \underline{x^3 - 5x^2} \\ 0 + x^2 - 2x \\ \quad + x^2 - 5x \\ \quad \underline{ + x^2 - 5x} \\ \quad 0 + 3x - 15 \\ \quad \quad + 3x - 15 \\ \quad \quad \underline{ + 3x - 15} \\ \quad \quad \quad 0 + 0 \end{array}}$$

$$\begin{array}{r} 0 + x^2 - 2x \\ + x^2 - 5x \\ \hline 0 + 3x - 15 \\ + 3x - 15 \\ \hline 0 + 0 \end{array}$$

$$(x-5)(x^2 + x + 3) = 0$$

$$\downarrow \quad \uparrow \quad b^2 - 4ac < 0 \\ x=5 \quad \therefore \text{no real roots}$$

$$1^2 - 4(1)(3) = -11$$

$$\textcircled{1} \quad x+1 \overline{) \begin{array}{r} x^2 - x + 1 \\ x^3 + 0x^2 + 0x + 1 \\ \underline{x^3 + x^2} \\ 0 - x^2 + 0x \\ \quad - x^2 - x \\ \quad \underline{ - x^2 - x} \\ \quad 0 + x + 1 \\ \quad \quad + x + 1 \\ \quad \quad \underline{ + x + 1} \\ \quad \quad \quad 0 + 0 \end{array}}$$

$$\therefore (x+1)(x^2 - x + 1)$$

$$\textcircled{2} \quad x+4 \overline{) \begin{array}{r} x^2 - 2x - 3 \\ x^3 + 2x^2 - 11x - 12 \\ \underline{x^3 + 4x^2} \\ 0 - 2x^2 - 11x \\ \quad - 2x^2 - 8x \\ \quad \underline{ - 2x^2 - 8x} \\ \quad 0 - 3x - 12 \\ \quad \quad - 3x - 12 \\ \quad \quad \underline{ - 3x - 12} \\ \quad \quad \quad 0 + 0 \end{array}}$$

$$\begin{array}{r} 0 - 2x^2 - 11x \\ - 2x^2 - 8x \\ \hline 0 - 3x - 12 \\ - 3x - 12 \\ \hline 0 + 0 \end{array}$$

$$(x+4)(x^2 - 2x - 3) \equiv (x+4)(x-3)(x+1)$$

\textcircled{b} $x > 3$ as a length cannot be $0 \geq$

$$\textcircled{3} \quad x^2+1 \overline{) \begin{array}{r} x^2 - 1 \\ x^4 + 0x^3 + 0x^2 - 1 \\ \underline{x^4 + 0x^3 + x^2} \\ 0 + 0 - x^2 - 1 \\ \quad - x^2 - 1 \end{array}}$$

$$\therefore (x^2+1)(x^2-1) \equiv (x^2+1)(x+1)(x-1)$$

37 The Factor Theorem

1) a) If $(x+1)$ is a factor
The $f(-1) = 0$.

$$f(-1) = (-1)^3 - 2(-1)^2 - 13(-1) - 10$$

$$= -1 - 2 + 13 - 10$$

$$= 0 \checkmark$$

b) $f(2) = (2)^3 - 2(2)^2 - 13(2) - 10$

$$= 8 - 8 - 26 - 10$$

$$= -36 \therefore \text{not}$$

c) $(x+1)(x-5)(x+2)$

2) If $(x-3)$ is a factor
a) $g(3) = 0$

$$g(3) = 2(3)^3 + (3)^2 + 3p + 12$$

$$0 = 54 + 9 + 3p + 12$$

$$0 = 75 + 3p$$

$$3p = -75$$

$$p = \frac{-75}{3}$$

$$\therefore p = -25$$

b) $x-3 \overline{) 2x^2 + 7x - 4}$

$$\begin{array}{r} 2x^3 + x^2 - 25x + 12 \\ 2x^3 - 6x^2 \\ \hline 0 + 7x^2 - 25x \\ + 7x^2 - 21x \\ \hline 0 - 4x + 12 \\ - 4x + 12 \\ \hline 0 + 0 \end{array}$$

$\therefore (x-3)(2x^2 + 7x - 4)$
 $(x-3)(2x-1)(x+4)$

c) $x=3, \frac{1}{2}$ or -4

3) $(x-3)$ is not a factor
a) of $h(x)$

b) $(3x+1)$ is not a factor
of $h(x)$.

1) $f(1) = 1^3 - 2(1) - 5(1) + 6$

$$= 1 - 2 - 5 + 6$$

$$= 0 \checkmark$$

a) $\therefore (x-1)$ is a factor.
NB: You can try $\pm 1, \pm 2, \pm 3, \pm 6$.

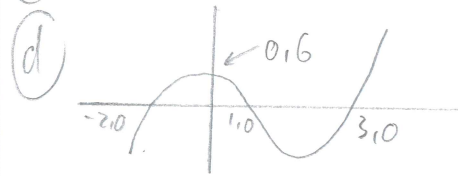
b) $x^2 - x - 6$

$$x-1 \overline{) x^3 - 2x^2 - 5x + 6}$$

$$\begin{array}{r} x^3 - x^2 \\ \hline 0 - x^2 - 5x \\ -x^2 + x \\ \hline 0 - 6x + 6 \\ -6x + 6 \\ \hline 0 + 0 \end{array}$$

$\therefore (x-1)(x^2 - x - 6)$
 $(x-1)(x+2)(x-3)$

c) $x=1, x=-2, x=3$



2) If $(x+2)$ is a factor then
 $f(-2) = 0$:

$$0 = 4(-2)^3 + p(-2)^2 + q(-2) - 12$$

$$0 = -32 + 4p - 2q - 12$$

$$2p - q = 22 \quad \text{1}$$

$f(-\frac{1}{4}) = 0$ too if $(4x+1)$ a factor

$$0 = 4(-\frac{1}{4})^3 + p(-\frac{1}{4})^2 + q(-\frac{1}{4}) - 12$$

$$0 = -\frac{1}{16} + \frac{p}{16} - \frac{q}{4} - 12$$

$$0 = -1 + p - 4q - 192$$

$$193 = p - 4q \quad \text{2}$$

Simultaneous equations

1) $2p - q = 22$

2) $p - 4q = 193$

1) $8p - 4q = 88$

2) $-1 - 7p = 105$

$$p = \frac{105}{-7}$$

$$= -15$$

1) $2(-15) - q = 22$

$$-30 - 22 = q$$

$$q = -52$$

b) $x-6$

$$(4x^2 + 9x + 2) \overline{) 4x^3 - 15x^2 - 52x - 12}$$

$$\begin{array}{r} 4x^3 + 9x^2 + 2x \\ \hline 0 - 24x^2 - 54x - 12 \\ -24x^2 - 54x - 12 \\ \hline 0 + 0 + 0 \end{array}$$

$\therefore (4x+1)(x+2)(x-6)$

37 continued.

$$① f(1) = a(1)^3 + b(1)^2 + c(1) - 2$$

$$\underline{0 = a + b + c - 2} \quad ①$$

$$f\left(-\frac{2}{3}\right) = a\left(-\frac{8}{27}\right) + b\left(\frac{4}{9}\right) + c\left(-\frac{2}{3}\right) - 2$$

$$0 = -8a + 12b - 18c - 54$$

$$\underline{0 = -4a + 6b - 9c - 27} \quad ②$$

$$f(2) = a(8) + b(4) + c(2) - 2$$

$$40 = 8a + 4b + 2c - 2$$

$$\underline{0 = 4a + 2b + c - 21} \quad ③$$

$$② + ③ \quad 0 = 8b - 8c - 48$$

$$① + ② \quad 0 = 10b - 5c - 35$$

$$6 = b - c \quad ④$$

$$7 = 2b - c \quad ⑤$$

$$\therefore \underline{b = 1} \quad \underline{c = -5}$$

$$0 = a + 1 - 5 - 2$$

$$\underline{\underline{a = 6}}$$

$$② \quad g(3) = (3)^4 + (3)^3 - 6(3)^2 + 6(3) - 72$$

$$= 81 + 27 - 54 + 18 - 72$$

$$① \quad = 0 \quad \checkmark$$

$$x-3 \overline{\begin{array}{r} x^3 + 4x^2 + 6x + 24 \\ x^4 + x^3 - 6x^2 + 6x - 72 \end{array}}$$

$$\underline{x^4 - 3x^3}$$

$$0 + 4x^2 - 6x^2$$

$$\underline{+ 4x^2 - 12x^2}$$

$$0 + 6x^2 + 6x$$

$$\underline{+ 6x^2 - 18x}$$

$$0 + 24x - 72$$

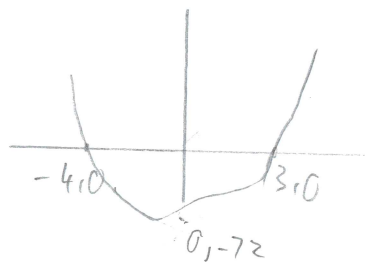
$$\underline{+ 24x - 72}$$

$$0 + 0$$

⑥

$$(x^3 + 4x^2 + 6x + 24)(x-3)$$

⑦ Quadratic with roots
(3,0) and (-4,0)



Steve Blades

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38 Mathematical Proof Introduction

① Expand the brackets

$$\begin{aligned} & (x-3)(2x+1)(2x+1) \\ & (x-3)(4x^2+4x+1) \\ & = 4x^3 + 4x^2 + x \\ & \quad - 12x^2 - 12x - 3 \\ & = 4x^3 - 8x^2 - 11x - 3 \end{aligned}$$

② Complete the square

$$\begin{aligned} & (x+1)^2 - 1 + 6 \\ & (x+1)^2 + 5 \end{aligned}$$

We know $(x+1)^2 \geq 0$
for all values of x
 $\therefore (x+1)^2 + 5 \geq 5$
for all values of x .

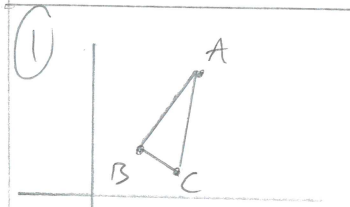
③ Rationalise the denominator.

$$\frac{x(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$$

$$\frac{x(2-\sqrt{3})}{4-2\sqrt{3}+2\sqrt{3}-3}$$

$$\frac{x(2-\sqrt{3})}{1}$$

$$x(2-\sqrt{3})$$



If $\triangle ABC$ is a right angle AB and BC will be perpendicular.

$$M_{AB} \times M_{BC} = -1$$

(or $M_{AB} = -\frac{1}{M_{BC}}$)

$$M_{AB} = \frac{8-4}{6-2} = \frac{4}{4} = 1$$

$$M_{BC} = \frac{4-3}{2-3} = \frac{1}{-1} = -1$$

$|x-1| = -1 \therefore$
 $\triangle ABC$ is a right angle

② Complete the square

$$\begin{aligned} & (x+2)^2 - 4 + C \\ & (x+2)^2 + C - 4 \end{aligned}$$

The function will be at its minimum when $x = -2$ and $y = C - 4 \therefore$
min point $(-2, C-4)$
N.B: you could differentiate and set to 0.



If $y = 3x + c$ is a chord it will have 2 points of intersection \therefore 2 roots to the simultaneous equations $y = 3x + c$ and $x^2 + y^2 = 36$.
The discriminant

$b^2 - 4ac > 0$ for 2 real roots

$$\begin{aligned} & y = 3x + c \quad \text{①} \\ & x^2 + y^2 = 36 \quad \text{②} \\ & x^2 + (3x+c)^2 = 36 \quad \text{③} \\ & x^2 + 9x^2 + 6cx + c^2 - 36 = 0 \quad \text{④} \\ & 10x^2 + 6cx + c^2 - 36 = 0 \\ & a = 10, b = 6c, c = c^2 - 36 \\ & (6c)^2 - 4(10)(c^2 - 36) > 0 \\ & 36c^2 - 40(c^2 - 36) > 0 \\ & 9c^2 - 10(c^2 - 36) > 0 \\ & 9c^2 - 10c^2 + 360 > 0 \\ & c^2 - 360 < 0 \\ & (c + 6\sqrt{10})(c - 6\sqrt{10}) < 0 \\ & \therefore -6\sqrt{10} < c < 6\sqrt{10} \end{aligned}$$

② If $\angle C = 90^\circ$
 M_{AB} and M_{BC} are perpendicular.
 $A(a,b)$, $B(b,d)$ and $C(e,d)$
 BC lies on the line $y = d \therefore AB$ must lie on the line $x = p$ where p is a constant.
 $p = a$ and $p = c \therefore a = c$

Pure (39) Methods of proof

(1) Proof by counter example:

$$3-2=1$$

(2) If n is odd one more than any odd is even $\therefore n+1$ is always even

(3) When $n=2$

$$2(2)^2+1=9$$

9 is not prime as it has 3 factors

(1)(a) let $2n$ be the first even number and $2n+2$ be the next even number.

$$(2n)^2 + (2n+2)^2 =$$

$$4n^2 + 4n^2 + 4n + 4n + 4$$
$$8n^2 + 8n + 4 \quad \text{Q.E.D.}$$

(b) $4(2n^2 + 2n + 1)$

The expression is divisible by 4 with no remainder \therefore a multiple of 4

(2) let n be the first integer and $(n+1)$ the second

$$(n+1)^3 - n^3 =$$

$$n^3 + 3n^2 + 3n + 1 - n^3$$
$$= 3n^2 + 3n + 1$$
$$= 3(n^2 + n) + 1$$

$3(n^2 + n)$ is a multiple of 3 $\therefore 3(n^2 + n) + 1$ is one more than a multiple of 3.

(1) My notes

$$\frac{a^2 + b^2}{2ab} \geq 1$$

$$a^2 + b^2 \geq 2ab$$

$$a^2 - 2ab + b^2 \geq 0$$

$$(a-b)^2 \geq 0$$

Proof:

We know that

$$(a-b)^2 \geq 0 \text{ for all real values}$$

$$a^2 - 2ab + b^2 \geq 0$$

$$a^2 + b^2 \geq 2ab$$

$$\frac{a^2 + b^2}{2ab} \geq 1 \quad \text{Q.E.D.}$$

\therefore True for all real positive values a and b .

(2) My notes

$$\frac{x}{y} + \frac{y}{x} \geq 2$$

$$\frac{y^2 + x^2}{xy} \geq 2$$

$$y^2 + x^2 \geq 2xy$$

$$x^2 - 2xy + y^2 \geq 0$$

$$(x-y)^2 \geq 0$$

Proof:

We know $(x-y)^2 \geq 0$ for all real values x and y .

$$(x-y)^2 \geq 0$$

$$x^2 - 2xy + y^2 \geq 0$$

$$x^2 + y^2 \geq 2xy$$

dividing both sides by xy

$$\frac{x^2}{xy} + \frac{y^2}{xy} \geq 2$$

$$\frac{x}{y} + \frac{y}{x} \geq 2 \quad \text{or} \quad \frac{y}{x} + \frac{x}{y} \geq 2 \quad \text{Q.E.D.}$$

\therefore True for all real positive values of x and y .

40 Binomial Expansion (using Pascal's Triangle)

① 1, 4, 6, 4, 1

$$1 \binom{4}{0} x^0 + 4 \binom{4}{1} x^1 + 6 \binom{4}{2} x^2 + 4 \binom{4}{3} x^3 + 1 \binom{4}{4} x^4$$

$$1(16)(1) + 4(8)(x) + 6(4)(x^2) + 4(2)(x^3) + 1(1)(x^4)$$

$$16 + 32x + 24x^2 + 8x^3 + x^4$$

② 1, 5, 10, 10, 5, 1

$$10 \binom{5}{1} (-2x)^1$$

$$10 \binom{5}{2} (-2x)^2$$

$$-80x^3$$

③ $(1+x) [1 \binom{4}{0} x^0 + 4 \binom{4}{1} x^1 + \dots]$

$$(1+x)(16 + 32x + \dots) = 1(32x) + x(16)$$

$$= 32x + 16x$$

$$= 48x \checkmark$$

④ 1, 5, 10, 10, 5, 1

$$10 \binom{5}{1} (-2x)^1$$

$$10 \binom{5}{2} (4x^2)$$

$$(270)(4x^2) = 1080x^2 \checkmark$$

② $(1+x) [1 \binom{3}{0} (-x)^0 + 3 \binom{3}{1} (-x)^1 + 3 \binom{3}{2} (-x)^2]$

$$(1+x)(1 - 3x + 3x^2)$$

$$1(3x^2) + x(-3x) = 3x^2 - 3x^2$$

$$= 0 \checkmark$$

③ 1, 4, 6, 4, 1

$$6 \binom{4}{2} (2x)^2 = 216x^2$$

$$(6a^2)(4x^2) = 216x^2$$

$$24a^2 = 216$$

$$a^2 = 9$$

$$a = \pm 3$$

④ $(1 + \frac{1}{x}) [1 \binom{4}{0} x^0 + 4 \binom{4}{1} x^1 + 6 \binom{4}{2} x^2 + 4 \binom{4}{3} x^3 + \dots]$

$$(1 + \frac{1}{x})(1 + 4x + 6x^2 + 4x^3 + \dots)$$

$$1(6x^2) + \frac{1}{x}(4x^3)$$

$$6x^2 + 4x^2 = 10x^2 \checkmark$$

② $1(p)^4(qx)^0 + 4(p)^3(qx)^1$

$$p^4 = 81 \quad \text{and} \quad 4pq^3 = \frac{4}{9}$$

$$p = \pm 3$$

$$p = 3 \text{ ①}$$

$$as p > 0 \quad pq^3 = \frac{1}{9} \text{ ②}$$

Simultaneous Equations

① into ②

$$3q^3 = \frac{1}{9} \quad \therefore p = 3, q = \frac{1}{3}$$

$$q^3 = \frac{1}{27}$$

$$q = \frac{1}{3}$$

③ $1 \binom{4}{0} (\sqrt{2})^0 + 4 \binom{4}{1} (\sqrt{2})^1 + 6 \binom{4}{2} (\sqrt{2})^2 + 4 \binom{4}{3} (\sqrt{2})^3 + 1 \binom{4}{4} (\sqrt{2})^4$

$$1(1)(1) + 4(1)(\sqrt{2}) + 6(1)(2) + 4(1)(2\sqrt{2}) + 1(1)(4)$$

$$1 + 4\sqrt{2} + 12 + 8\sqrt{2} + 4 =$$

$$17 + 12\sqrt{2} \quad \therefore a = 17, b = 17$$

⑥ The odd powers will be negative in $(1 - \sqrt{2})^4$ \therefore when you add the two the $\sqrt{2}$'s will cancel to leave

$$(1+1) + (12+12) + (4+4) = 34 \text{ or } 2a$$