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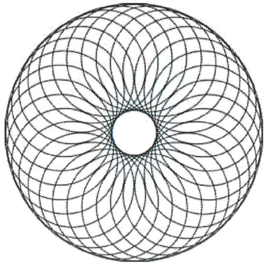
A LEVEL MATHS

YEAR 1 PURE



ANSWERS

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Pure ㊱ Dividing Polynomials

①
$$\begin{array}{r|l} & x^2+3x-18 \\ \hline x+1 & x^3+4x^2-15x-18 \\ & \underline{x^3+x^2} \\ & 0+3x^2-15x \\ & \underline{+3x^2+3x} \\ & 0-18x-18 \\ & \underline{-18x-18} \\ & 0+0 \end{array}$$

No remainder. → $0+0$

②
$$\begin{array}{r|l} & x^2+11x+30 \\ \hline x+2 & x^3+13x^2+52x+60 \\ & \underline{x^3+2x^2} \\ & 0+11x^2+52x \\ & \underline{+11x^2+22x} \\ & +0+30x+60 \\ & \underline{+30x+60} \\ & 0+0 \end{array}$$

ⓐ

ⓑ $x^3+13x^2+52x+60 \equiv$

$(x+2)(x^2+11x+30)$

$(x+2)(x+6)(x+5)$

③
$$\begin{array}{r|l} & x^2+6x+2 \\ \hline x-3 & x^3+3x^2-16x+7 \\ & \underline{x^3-3x^2} \\ & 0+6x^2-16x \\ & \underline{+6x^2-18x} \\ & 0+2x+7 \\ & \underline{+2x-6} \\ & 0+13 \end{array}$$

$r=13$

①
$$\begin{array}{r|l} & x^2-x-6 \\ \hline x+1 & x^3+0x^2-7x-6 \\ & \underline{x^3+x^2} \\ & 0-x^2-7x \\ & \underline{-x^2-x} \\ & 0-6x-6 \\ & \underline{-6x-6} \\ & 0+0 \end{array}$$

ⓐ

ⓑ $(x+1)(x^2-x-6)$

$(x+1)(x-3)(x+2)$

②

② ⓐ
$$\begin{array}{r|l} & x^2+6x-7 \\ \hline 2x+1 & 2x^3+13x^2-8x-7 \\ & \underline{2x^3+x^2} \\ & 0+12x^2-8x \\ & \underline{+12x^2+6x} \\ & 0-14x-7 \\ & \underline{-14x-7} \\ & 0+0 \end{array}$$

ⓑ $g(x) \equiv (2x+1)(x^2+6x-7)$

$0=(2x+1)(x+7)(x-1)$

$\therefore x=-\frac{1}{2}, x=-7\ x=1$

③
$$\begin{array}{r|l} & x^2+x+3 \\ \hline x-5 & x^3-4x^2-2x-15 \\ & \underline{x^3-5x^2} \\ & 0+x^2-2x \\ & \underline{+x^2-5x} \\ & 0+3x-15 \\ & \underline{+3x-15} \\ & 0+0 \end{array}$$

$(x-5)(x^2+x+3)=0$

↓ $x=5$

↑ $b^2-4ac<0$ ∴ no real roots

$1^2-4(1)(3)=-11$

①
$$\begin{array}{r|l} & x^2-x+1 \\ \hline x+1 & x^3+0x^2+0x+1 \\ & \underline{x^3+x^2} \\ & 0-x^2+0x \\ & \underline{-x^2-x} \\ & 0+x+1 \\ & \underline{+x+1} \\ & 0+0 \end{array}$$

$\therefore (x+1)(x^2-x+1)$

②
$$\begin{array}{r|l} & x^2-2x-3 \\ \hline x+4 & x^3+2x^2-11x-12 \\ & \underline{x^3+4x^2} \\ & 0-2x^2-11x \\ & \underline{-2x^2-8x} \\ & 0-3x-12 \\ & \underline{-3x-12} \\ & 0+0 \end{array}$$

$(x+4)(x^2-2x-3) \equiv (x+4)(x-3)(x+1)$

ⓑ $x>3$ as a length cannot be $0 \geq$

③
$$\begin{array}{r|l} & x^2-1 \\ \hline x^2+0x+1 & x^4+0x^3+0x^2-1 \\ & \underline{x^4+0x^3+x^2} \\ & 0+0-x^2-1 \\ & \underline{-x^2-1} \end{array}$$

$\therefore (x^2+1)(x^2-1) \equiv (x^2+1)(x+1)(x-1)$

㊲ The Factor Theorem

① ⓐ If $(x+1)$ is a factor

The $f(-1)=0$.

$f(-1)=(-1)^3-2(-1)^2-13(-1)-10$

$=-1-2+13-10$

$=0$ ✓

ⓑ $f(2)=(2)^3-2(2)^2-13(2)-10$

$=8-8-26-10$

$=-36$ ∴ not

ⓒ $(x+1)(x-5)(x+2)$

② ⓐ If $(x-3)$ is a factor

$g(3)=0$

$g(3)=2(3)^3+(3)^2+3p+12$

$0=54+9+3p+12$

$0=75+3p$

$3p=-75$

$p=\frac{-75}{3}$

∴ $p=-25$

ⓑ

$$\begin{array}{r|l}
 & 2x^2+7x-4 \\ \hline
x-3 & 2x^3+x^2-25x+12 \\
 & 2x^3-6x^2 \\ \hline
 & 0\quad +7x^2-25x \\
 & +7x^2-21x \\ \hline
 & 0\quad -4x+12 \\
 & -4x+12 \\ \hline
 & 0+0
\end{array}$$

∴ $(x-3)(2x^2+7x-4)$

$(x-3)(2x-1)(x+4)$

ⓒ $x=3,\ \frac{1}{2}$ or -4

③ ⓐ $(x-3)$ is not a factor of $h(x)$

ⓑ $(3x+1)$ is not a factor of $h(x)$.

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① ⓐ $f(1)=1^3-2(1)-5(1)+6$

$=1-2-5+6$

$=0$ ✓

∴ $(x-1)$ is a factor.

N.B: You can try ±1, ±2 ±3, ±6.

ⓑ

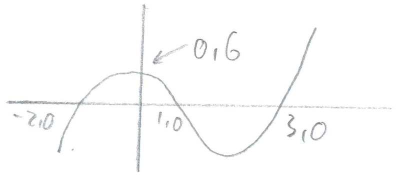
$$\begin{array}{r|l}
 & x^2-x-6 \\ \hline
x-1 & x^3-2x^2-5x+6 \\
 & x^3-x^2 \\ \hline
 & 0\quad -x^2-5x \\
 & -x^2+x \\ \hline
 & 0\quad -6x+6 \\
 & -6x+6 \\ \hline
 & 0+0
\end{array}$$

∴ $(x-1)(x^2-x-6)$

$(x-1)(x+2)(x-3)$

ⓒ $x=1,\ x=-2,\ x=3$

ⓓ



② If $(x+2)$ is a factor then

$f(-2)=0$:

$0=4(-2)^3+p(-2)^2+q(-2)-12$

$0=-32+4p-2q-12$

$2p-q=22$ ①

$f(-\frac{1}{4})=0$ too if $(4x+1)$ a factu

$0=4(-\frac{1}{4})^3+p(-\frac{1}{4})^2+q(-\frac{1}{4})-12$

$0=-\frac{1}{16}+\frac{p}{16}-\frac{q}{4}-12$

$0=-1+p-4q-192$

$193=p-4q$ ②

Simultaneous equations

① $2p-q=22$

② $p-4q=193$

① $8p-4q=88$

② − ① $-7p=105$

$p=\frac{105}{-7}$

$=-15$

① $2(-15)-q=22$

$-30-22=q$

$q=-52$

ⓑ

$$\begin{array}{r|l}
 & x-6 \\ \hline
(4x^2+9x+2) & 4x^3-15x^2-52x-12 \\
 & 4x^3+9x^2+2x \\ \hline
 & 0\quad -24x^2-54x-12 \\
 & -24x^2-54x-12 \\ \hline
 & 0+0+0
\end{array}$$

∴ $(4x+1)(x+2)(x-6)$

㊲ Continued

① $f(1) = a(1)^3 + b(1)^2 + c(1) - 2$

$0 = a + b + c - 2$ ①

$f\left(-\frac{2}{3}\right) = a\left(-\frac{8}{27}\right) + b\left(\frac{4}{9}\right) + c\left(\frac{-2}{3}\right) - 2$

$0 = -8a + 12b - 18c - 54$

$0 = -4a + 6b - 9c - 27$ ②

$f(2) = a(8) + b(4) + c(2) - 2$

$40 = 8a + 4b + 2c - 2$

$0 = 4a + 2b + c - 21$ ③

②+③ $0 = 8b - 8c - 48$

①+② $0 = 10b - 5c - 35$

$6 = b - c$ ④

$7 = 2b - c$ ⑤

$\therefore b = 1 \quad c = -5$

$0 = a + 1 - 5 - 2$

$a = 6$

② $g(3) = (3)^4 + (3)^3 - 6(3)^2 + 6(3) - 72$

ⓐ $= 81 + 27 - 54 + 18 - 72$

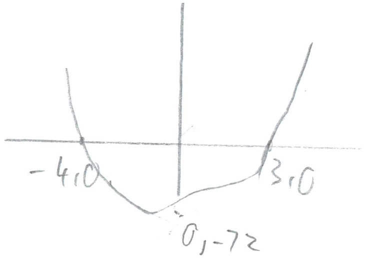
$= 0$ ✓

$$\begin{array}{r|l} & x^3 + 4x^2 + 6x + 24 \\ \hline x-3 & x^4 + x^3 - 6x^2 + 6x - 72 \\ & x^4 - 3x^3 \\ \hline & 0 + 4x^3 - 6x^2 \\ & +4x^3 - 12x^2 \\ \hline & 0 + 6x^2 + 6x \\ & +6x^2 - 18x \\ \hline & 0 \quad + 24x - 72 \\ & +24x - 72 \\ \hline & 0 + 0 \end{array}$$

ⓑ

$(x^3 + 4x^2 + 6x + 24)(x - 3)$

ⓒ Quartic with roots (3,0) and (-4,0)



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㊳ Mathematical Proof Introduction

① Expand the brackets

$(x-3)(2x+1)(2x+1)$

$(x-3)(4x^2+4x+1)$

$\equiv 4x^3+4x^2+x$
$-12x^2-12x-3$

$= 4x^3-8x^2-11x-3$

② Complete the square

$(x+1)^2-1+6$

$(x+1)^2+5$

We know $(x+1)^2 \geq 0$ for all values of x

∴ $(x+1)^2+5 \geq 5$ for all values of x.

③ Rationalise the denominator.

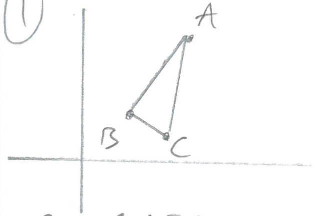
$$\frac{x(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$$

$$\frac{x(2-\sqrt{3})}{4-\cancel{2\sqrt{3}}+\cancel{2\sqrt{3}}-3}$$

$$\frac{x(2-\sqrt{3})}{1}$$

$$x(2-\sqrt{3})$$

①



If $<ABC$ is a right angle AB and BC will be perpendicular.

$M_{AB} \times M_{BC} = -1$

(or $M_{AB} = -\frac{1}{M_{BC}}$)

$M_{AB} = \frac{8-4}{6-2} = \frac{4}{4} = 1$

$M_{BC} = \frac{4-3}{2-3} = \frac{1}{-1} = -1$

$1 \times -1 = -1$ ∴

$<ABC$ is a right angle

② Complete the square

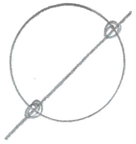
$(x+2)^2-4+c$

$(x+2)^2+c-4$

The function will be at its minimum when $x=-2$ and $y=c-4$ ∴ min point $(-2, c-4)$

N.B: you can differentiate and set to 0.

①



If $y=3x+c$ is a chord it will have 2 points of intersection ∴ 2 roots to the simultaneous equations $y=3x+c$ and $x^2+y^2=36$. The discriminant

$b^2-4ac>0$ for 2 real roots

$y=3x+c$ ①

$x^2+y^2=36$ ②

$x^2+(3x+c)^2=36$ ②

$x^2+9x^2+6cx+c^2-36=0$ ②

$10x^2+6cx+c^2-36=0$

$a=10,\ b=6c,\ c=c^2-36$

$(6c)^2-4(10)(c^2-36)>0$

$36c^2-40(c^2-36)>0$

$9c^2-10(c^2-36)>0$

$9c^2-10c^2+360>0$

$c^2-360<0$

$(c+6\sqrt{10})(c-6\sqrt{10})<0$

∴ $-6\sqrt{10}<c<6\sqrt{10}$

② If $x=90°$

M_{AB} and M_{BC} are perpendicula.

$A(a,b)$, $B(b,d)$ and $C(e,d)$

BC lies on the line $y=d$ ∴ AB must lie on the line $x=p$ where p is a constant.

$p=a$ and $p=c$ ∴ $a=c$

Pure ㊴ Methods of proof

① Proof by counter example:

$3 - 2 = 1$

② If n is odd one more than any odd is even ∴ n+1 is always even

③ when $n=2$

$2(2)^2+1 = 9$

9 is not prime as it has 3 factors

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①ⓐ let 2n be the first even number and 2n+2 be the next even number.

$(2n)^2+(2n+2)^2 =$

$4n^2+4n^2+4n+4n+4$

$8n^2+8n+4$ Q.E.D

ⓑ $4(2n^2+2n+1)$

The expression is divisible by 4 with no remainder ∴ a multiple of 4

② let n be the first integer and (n+1) the second

$(n+1)^3 - n^3 =$

$n^3+3n^2+3n+1-n^3$

$= 3n^2+3n+1$

$= 3(n^2+n)+1$

$3(n^2+n)$ is a multiple of 3 ∴ $3(n^2+n)+1$ is one more than a multple of 3.

① My notes

$\frac{a^2+b^2}{2ab} \geq 1$

$a^2+b^2 \geq 2ab$

$a^2-2ab+b^2 \geq 0$

$(a-b)^2 \geq 0$

Proof:

We know that $(a-b)^2 \geq 0$ for all real values

$a^2-2ab+b^2 \geq 0$

$a^2+b^2 \geq 2ab$

$\frac{a^2+b^2}{2ab} \geq 1$ Q.E.D

∴ True for all real positive values a and b.

② My notes

$\frac{y}{x} + \frac{x}{y} \geq 2$

$\frac{y^2+x^2}{xy} \geq 2$

$y^2+x^2 \geq 2xy$

$x^2-2xy+y^2 \geq 0$

$(x-y)^2 \geq 0$

Proof:

We know $(x-y)^2 \geq 0$ for all real values x and y.

$(x-y)^2 \geq 0$

$x^2-2xy+y^2 \geq 0$

$x^2+y^2 \geq 2xy$

dividing both sides by xy

$\frac{x^2}{xy} + \frac{y^2}{xy} \geq 2$

$\frac{x}{y} + \frac{y}{x} \geq 2$ or $\frac{y}{x} + \frac{x}{y} \geq 2$ Q.E.D

∴ True for all real postive values of x and y.

㊵ Binomial Expansion (Using Pascals Triangle)

① 1,4,6,4,1

$1(2)^4(x)^0+4(2)^3(x)^1+6(2)^2(x)^2+4(2)^1(x)^3+1(2)^0(x)^4$

$1(16)(1)+4(8)(x)+6(4)(x^2)+4(2)(x^3)+1(1)(x^4)$

$16+32x+24x^2+8x^3+x^4$

② 1,5,10,10,5,1

$10(1)^2(-2x)^3$

$10(1)(-8x^3)$

$-80x^3$

③ $(1+x)\left[1(2)^4(x)^0+4(2)^3(x)^1+\dots\right]$

$(1+x)(16+32x+\dots) \equiv 1(32x)+x(16)$

$= 32x+16x$

$= 48x$ ✓

① 1,5,10,10,5,1

$10(3)^3(-2x)^2$

$10(27)(4x^2)$

$(270)(4x^2)=1080x^2$ ✓

② $(1+x)\left[1(1)^3(-x)^0+3(1)^2(-x)^1+3(1)^1(-x)^2+\right]$

$(1+x)(1-3x+3x^2)$

$1(3x^2)+x(-3x) \equiv 3x^2-3x^2$

$= 0$ ✓

③ 1,4,6,4,1

$6(a)^2(2x)^2=216x^2$

$(6a^2)(4x^2)=216x^2$

$24a^2=216$

$a^2=9$

$a=\pm3$

① $(1+\frac{1}{x})\left[1(1)^4(x)^0+4(1)^3(x)+6(1)^2(x)^2+4(1)^1(x)^3+\dots\right]$

$(1+\frac{1}{x})\left[1+4x+6x^2+4x^3+\dots\right]$

$1(6x^2)+\frac{1}{x}(4x^3)$

$6x^2+4x^2 \equiv 10x^2$ ✓

② $1(p)^4(qx)^0+4(p)^1(qx)^3$

$p^4=81$ and $4pq^3=\frac{4}{9}$

$p=\pm3$ ②

$p=3$ ①

as $p>0$ $pq^3=\frac{1}{9}$ ②

Simultaneous Equations

① into ②

$3q^3=\frac{1}{9}$ ∴ $p=3, q=\frac{1}{3}$

$q^3=\frac{1}{27}$

$q=\frac{1}{3}$

③

$1(1)^4(\sqrt{2})^0+4(1)^3(\sqrt{2})^1+6(1)^2(\sqrt{2})^2+4(1)^1(\sqrt{2})^3+1(1)^0(\sqrt{2})^4$

$1(1)(1)+4(1)(\sqrt{2})+6(1)(2)+4(1)(2\sqrt{2})+1(1)(4)$

$1+4\sqrt{2}+12+8\sqrt{2}+4=$

$17+12\sqrt{2}$ ∴ $a=17, b=17$

⑥ The odd powers will be negative in $(1-\sqrt{2})^4$ ∴ when you add the two the $\sqrt{2}$'s will cancel to leave

$(1+1)+(12+12)+(4+4)=34$ or $2a$