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A LEVEL MATHS

YEAR 1 PURE

ANSWERS

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(1) Indices
(2) Expanding Brackets
(3) Factorising Expressions
(4) More Indices (Negative and Fractional)
(5) Working with Surds
(6) Solving Quadratic Equations
(7) Completing the Square for Quadratics Expressions
(8) Function Notation
(9) Sketching Quadratic Graphs
(10) The Discriminant for Quadratic Equations
(11) Applications of Quadratics Equations
(12) Solving Linear Simultaneous Equations
(13) Linear & Non-Linear Simultaneous Equations
(14) Graphing Simultaneous Equations
(15) Linear Inequalities
(16) Quadratic Inequalities
(17) Graphing Inequalities
(18) Shading Inequalities
(19) Cubic Graphs
(20) Quartic Graphs
(21) Reciprocal Graphs
(22) The Intersection of Graphs
(23) Transforming Graphs (Translations)
(24) Transforming Graphs (Stretching/Reflecting)
(25) Straight Line Graphs in the form $y = mx + c$
(26) More Straight Line Graphs
(27) Straight Line Graphs (Parallel & Perpendicular)
(28) The Geometry of Straight Lines
(29) The Application of Linear Graphs
(30) Circle Geometry Midpoint & Perpendicular
(31) The Equation of a Circle
(32) Circles and Straight Lines (Intersections)
(33) Circles (Tangents and Chords)
(34) Circles and Triangles
(35) Algebraic Fractions
(36) Polynomial Division
(37) The Factor and Remainder Theorem
(38) An Introduction to Mathematical Proof
(39) Methods of Proof
(40) Binomial Expansion (Using Pascal's Triangle)
(41) Binomial Expansion (Factorial Notation)
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(43) Binomial Expansion (Problem Solving)
(44) Binomial Expansion (Estimations and Approximations)
(45) The Cosine Rule
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(47) Areas of a Triangles
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(51) The 'CAST' Diagram for Trig Ratios
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(53) Proving Trigonometric Identities
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(56) Using Identities to Solve Trig Equations
(57) Vectors (Introduction)
(58) Vector Notation (Column and i and j form)
(59) Vectors (Magnitude and Direction)
(60) Vectors (Position and Direction Vectors)
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(62) Application of Vectors
(63) Differentiation (Gradients of Curves)
(64) Differentiation from 1st Principles
(65) Differentiating x^n (Basic Powers of)
(66) Differentiation (Quadratic Expression)
(67) Differentiation (Multiple Terms)
(68) Differentiation (Gradients, Tangents and Normals)
(69) Differentiation (Increasing and Decreasing Functions)
(70) Differentiation (Stationary Points)
(71) Differentiation (Gradient Functions
(72) The Applications of Differentiation
(73) Integration (Basic Expressions (x^n))
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(75) Integration (Finding c and Finding Functions)
(76) Integration (Definite Integrals)
(77) Integration (Basic Areas Under Curves)
(78) Integration ('Negative and Positive Areas')
(79) Integration (Areas between Curves and Lines)
(80) Basic Exponential Functions
(81) 'The' Exponential Function $y = e^x$
(82) Applications of Basic Exponential Models
(83) Logarithms (Simplifying & Evaluating)
(84) Logarithms (The Log Laws)
(85) Logarithms (Log and Exponential Equations)

31 Pure. Equation of a Circle

1. $x^2+y^2=36$
2. $(x+3)^2+(y-6)^2=49$
3. Complete the square

$(x+1)^2-1+(y-2)^2-4-20=0$

$(x+1)^2+(y-2)^2=25$

Centre $(-1,2)$

radius 5

---

1. $(0-4)^2+(2+p)^2=97$

$16+(2+p)^2=97$

$(2+p)^2=81$

$2+p=\pm 9$

$p=7$ or $p=-11$

---

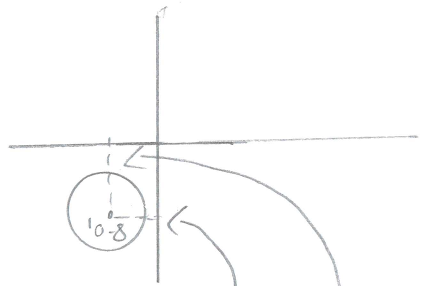
2. $\left(x+\frac{3}{2}\right)^2-\frac{9}{4}+\left(y-\frac{5}{2}\right)^2-\frac{25}{4}-2=0$

$\left(x+\frac{3}{2}\right)^2+\left(y-\frac{5}{2}\right)^2-\frac{34}{4}-2=0$

$\left(x+\frac{3}{2}\right)^2+\left(y-\frac{5}{2}\right)^2=\frac{42}{4}$

$\therefore r^2=\frac{42}{4}$, $r=\frac{\sqrt{42}}{2}$

3. Centre is $(8,-10)$ and radius is $\sqrt{60}\approx 7.75$



$8-7.75\approx 0.25$

$10-7.75\approx 2.25$

$\therefore$ doesn't cross

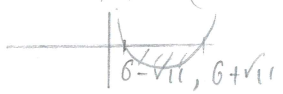
---

1. $(7-4)^2+(p-6)^2<(2\sqrt{5})^2$

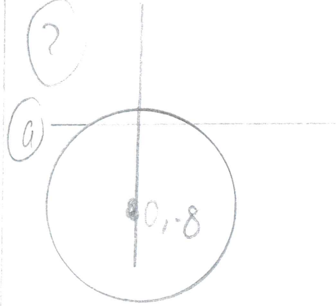
$9+p^2-12p+36-20<0$

$p^2-12p+25<0$

$(p-(6+\sqrt{11}))(p-(6-\sqrt{11}))<0$



$6-\sqrt{11}<p<6+\sqrt{11}$



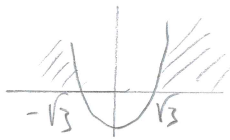
$r>8$

6. $y=2$, $y=-18$

3. $(x-3)^2-9+(y+p)^2-p^2+12=0$

$(x-3)^2+(y+p)^2=p^2-3$

$\therefore p^2-3>0$ for radius to exist



$p<-\sqrt{3}$ or $p>\sqrt{3}$

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Steve Blades

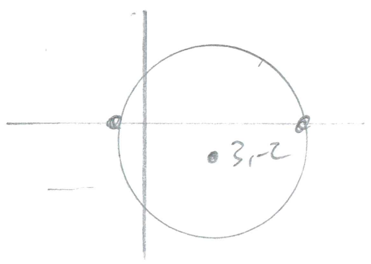
(32) Intersections of Lines and Circles

① $y=3$ ①
$x^2+y^2=25$ ②

$[x^2+(3)^2=25$ ②
$x^2+9=25$
$x^2=16$
$\therefore x=\pm 4$ and
$y=3$
Points $(-4,3)$ and $(4,3)$

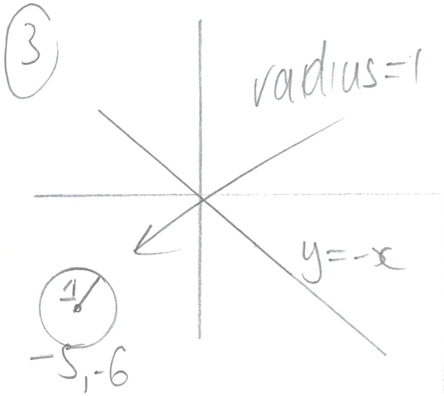
② $x=0$
$\therefore (-3)^2+(y+2)^2=45$
(a) $(9)+(y+2)^2=45$
$(y+2)^2=36$
$y+2=\pm 6$
$y=4$ or $y=-8$
$\therefore (0,4)$ or $(0,-8)$

(b)



The circle crosses the x axis when $y=0$
$\therefore (x-3)^2+(2)^2=45$
$(x-3)^2+4=45$
$(x-3)^2=41$
$x-3=\pm\sqrt{41}$
$x=3\pm\sqrt{41}$
$\therefore (3+\sqrt{41},0)$ and $(3-\sqrt{41},0)$

③



① A chord will have 2 points of intersection
$(x-4)^2+(x-3)^2=1$
$x^2-8x+16+x^2-6x+9=1$
$2x^2-14x+24=0$
$x^2-7x+12=0$
$(x-3)(x-4)=0$
$x=3$ or $x=4$
$\therefore (3,3)$ and $(4,4)$

② (a) centre $(-3,5)$
radius$=5$

(b) $3x-4y+29=0$
$4y=3x+29$
$y=\frac{3}{4}x+\frac{29}{4}$ ①
$(x+3)^2+(y-5)^2=25$ ②
$(x+3)^2+\left(\frac{3}{4}x+\frac{9}{4}\right)^2=25$
$x^2+6x+9+\frac{9}{16}x^2+\frac{54}{16}x+\frac{81}{16}=25$
$\frac{25}{16}x^2+\frac{150}{16}x=\frac{175}{16}$
$25x^2+150-175=0$
$x^2+6x-7=0$
$(x+7)(x-1)=0$
$x=1,\ x=-7$
$y=8,\ 2$
points: $(1,8)$ $(-7,2)$

(c) radius$=5$
$\therefore$ diameter$=10$
$(1--7)^2+(8-2)^2$
8^2+6^2
$64+36=100$
$\therefore$ length$=10$ ✓

③ $b^2-4ac=0$ for tangent:
$(x-3)^2+\left(\frac{4x-31}{3}-2\right)^2=25$
$x^2-6x+9+\left(\frac{4x-37}{3}\right)^2=25$
$x^2-6x+4+\frac{16x^2-296x+1369}{9}=25$
$9x^2-54x+81+16x^2-296x+1369=225$
$25x^2-350x+1225=0$
$x^2-14x+49=0$
$a=1,\ b=-14\ c=49$
$(-14)^2-4(1)(49)$
$196-196=0$ ✓

① If a tangent
$b^2-4ac=0$
$(x+2)^2+(mx+1)^2=1$
$x^2+4x+4+m^2x^2+2mx+1=1$
$x^2(1+m^2)+x(4+2m)+4=0$
$a=(1+m^2)$
$b=(4+2m)$
$c=4$
$(4+2m)^2-4(1+m^2)(4)=0$
$\cancel{16}+16m+4m^2-16m^2-\cancel{16}=0$
$16m-12m^2=0$
$4m(4m-3)=0$
$m=0,\ m=\frac{3}{4}$

(32) (Cont) Year1 Pure.
(Intersections of lines and circles)

(2) If a chord $b^2-4ac>0$

$y=\frac{1}{2}x+c$ ①

$2y=x+2c$

$2y-2c=x$ ①

$(2y-2c)^2+(y-8)^2=20$

$4y^2-8yc+4c^2+$

$y^2-16y+64=20$

$5y^2+y(-16-8c)$
$+4c^2+44=0$

$a=5$
$b=(-16-8c)$
$c=4c^2+44$

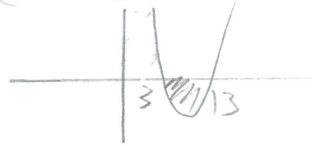
$(-16-8c)^2-4(5)(4c^2+44)>0$

$256+256c+64c^2-80c^2-880>0$

$-16c^2+256c-624>0$

$c^2-16c+39<0$

$(c-3)(c-13)<0$



$3<c<13$

(3) Circle

$(x+5)^2-25+(y-3)^2-9=66$

$(x+5)^2+(y-3)^2=100$

$y=-\frac{5}{2}x+\frac{27}{2}$

$\therefore (x+5)^2+\left(\frac{21}{2}-\frac{5}{2}x\right)^2=100$

$x^2+10x+25+\frac{441}{4}-\frac{210}{4}x+\frac{25}{4}x^2=100$

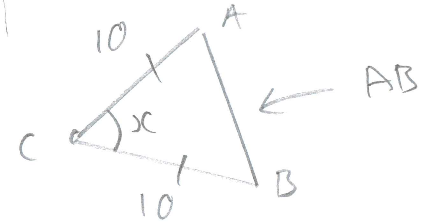
$4x^2+40x+100+441-210x+25x^2=400$

$29x^2-170x+141=0$

$x=1,\quad x=\frac{141}{29}$

$y=11\quad y=\frac{39}{29}$

A and B have coordinates $(1,11)$ and $\left(\frac{141}{24},\frac{39}{24}\right)$



$AB^2=(11-1)^2+\left(\frac{39}{24}-\frac{141}{29}\right)^2$

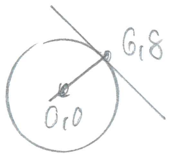
$AB^2\approx 112.37\ldots$

$\therefore \cos(x)=\frac{10^2+10^2-AB^2}{2(10)(10)}$

$\cos x=\frac{9212}{21025}$

33 Use Tangent and Chord Properties

(1)



$M_r = \frac{8-0}{6-0} = \frac{4}{3}$

$\therefore M_T = -\frac{3}{4}$

as M_r and M_T are $\perp$.

$y-8 = -\frac{3}{4}(x-6)$

$y = -\frac{3}{4}x + \frac{25}{2}$

(2) The radius is a $\perp$ bisector of a chord

$\therefore$ midpoint of AB

$\left(\frac{3+7}{2}, \frac{-1+7}{2}\right) = (5,3)$

(3) $-\frac{1}{m}$

(1) Centre $(3,4)$ $\therefore$

$(x-3)^2+(y-4)^2=r^2$

$r^2 = (4-0)^2+(3-0)^2$

$r^2 = 25$

$\therefore (x-3)^2+(y-4)^2=25$

(b) D is the midpoint of AB

$\therefore \left(\frac{3+7}{2}, \frac{-1+7}{2}\right) = \left(\frac{10}{2}, \frac{6}{2}\right)$

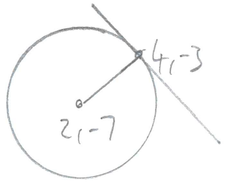
$= (5,3)$

(c) $CD = \sqrt{(5-3)^2+(3-4)^2}$

$= \sqrt{2^2+1^2}$

$= \sqrt{5}$

(2)



$M_r = \frac{-3--7}{4-2} = \frac{4}{2} = 2$

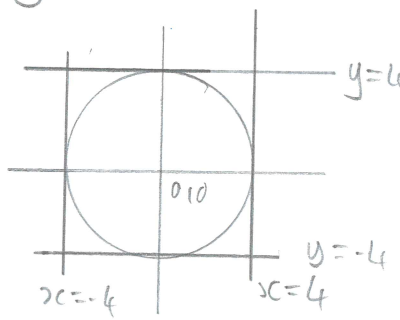
$\therefore M_T = -\frac{1}{2}$

$y--3 = -\frac{1}{2}(x-4)$

$2y+6 = -x+4$

$x+2y+2=0$

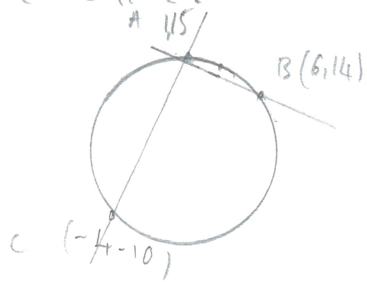
(3)



Vertical: $x=-4, x=4$

Horizontal: $y=-4, y=4$

(1) The $\perp$ bisector of a chord will pass through the centre.



$\perp$ bisector of AC

Midpoint: $\left(\frac{-4+6}{2}\right), \left(\frac{-10+14}{2}\right)$

$(1,2)$

Gradient of AC

$\frac{14--10}{6--4} = \frac{12}{5}$

$\therefore$ $\perp$ bisector of AC will have gradient $-\frac{5}{12}$

line

$y-2 = -\frac{5}{12}(x-1)$

$y = -\frac{5}{12}x - \frac{29}{12}$ (1)

$\perp$ bisector of AB

Midpoint $\left(\frac{1+6}{2}\right), \left(\frac{15+14}{2}\right) =$

$\frac{7}{2}, \frac{29}{2}$

Gradient of AB $= \frac{15-14}{1-6} = -\frac{1}{5}$

$\therefore$ Gradient of $\perp$ bisector $= 5$

line

$y - \frac{29}{2} = 5\left(x - \frac{7}{2}\right)$

$y = 5x - 3$ (2)

Simultaneous equations

$5x-3 = -\frac{5}{12}x - \frac{29}{12}$

$60x - 36 = -5x - 29$

$65x = 29+36$

$65x = 65$

$x = 1$

Sub in, $y = 5(1)-3$

$y=2$

$\therefore (1,2)$

(2) $\frac{13-8}{12-a} = +\frac{5}{7}$

$\frac{5}{12-a} = \frac{5}{7}$

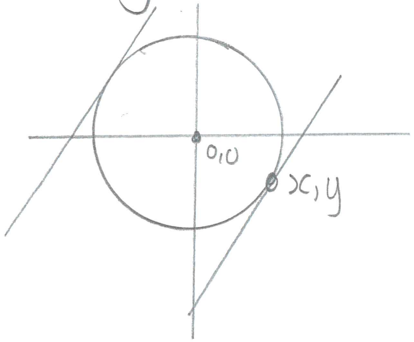
$35 = 60-5a$

$5a = 25$

$a = 5$

33 Continued.

3) $x^2+y^2=125$



$m_r = \pm\frac{x}{y}$

$m_T = 2$

$\therefore \pm\frac{x}{y} = \frac{-1}{2}$

or $y = 2x$

Sub in

$x^2+(2x)^2=125$

$5x^2=125$

$x^2=25$

$x=\pm 5$

$y=2x$

$\therefore y=\pm 10$

Points $(-5,10)$ and $(5,-10)$

34 Circles and Triangles

1 $(10)^2+(0)^2$

a $100+0$

100 ✓

b $r=10$

c If it's a diameter

AB = 20 units long.

$AB=\sqrt{(6--6)^2+(8--8)^2}$

$=\sqrt{12^2+16^2}$

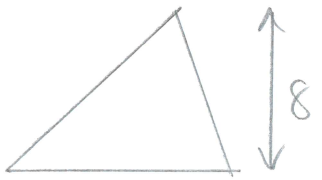
$=\sqrt{144+256}$

$=\sqrt{400}$

$=20$ ✓

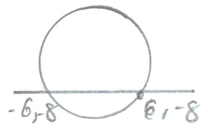
d $\tan^{-1}\left(\frac{8}{6}\right)=53.1^\circ$

e



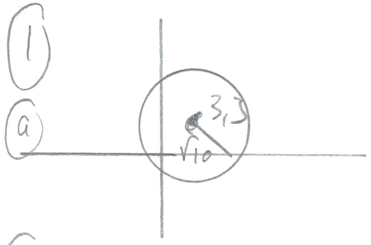
$\frac{10\times8}{2}=40u^2$

f $(6,-8)$.

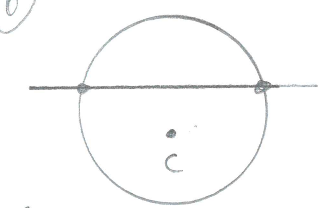


1

a



b



$(x-3)^2+(y-3)^2=10$ (1)

$y=4$ (2)

$(x-3)^2+(4-1)^2=10$

$(x-3)^2+1=10$

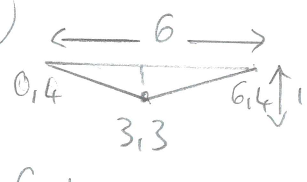
$(x-3)^2=9$

$(x-3)=\pm3$

$x=6, x=0$

$\therefore (6,4)$ and $(0,4)$

c



$\frac{6\times1}{2}=3u^2$.

2 Sub in $(1,1)$

a $(1-6)^2+(1+1)^2$

$(-5)^2+(2)^2$

$25+4=29$ ✓

Sub in $(4,4)$

$(4-6)^2+(4+1)^2$

$(-2)^2+5^2$

$4+25=29$ ✓

$\therefore$ Both lie on circle.

b If a diameter

$PQ=2\sqrt{29}$ (or $PQ^2=116$)

$(4-1)^2+(4-1)^2$

3^2+3^2

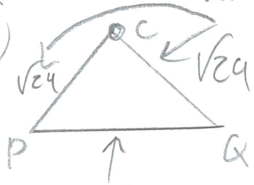
$9+9=18$

$\therefore$ Not a diameter.

MANY WAYS TO DO PART b

c $(6,-1)$

d



$\sqrt{18}=3\sqrt{2}$

$\therefore PC+CQ+PQ=$
$\sqrt{29}+\sqrt{29}+3\sqrt{2}$

$=2\sqrt{29}+3\sqrt{2}$

e distance of the point $(7,5)$ from $(6,-1)$ must be greater than $\sqrt{29}$ to be outside.

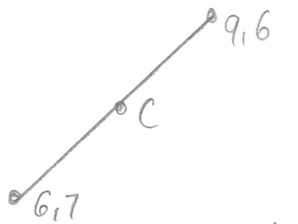
$\sqrt{(7-6)^2+(5--1)^2}$

$\sqrt{1+36}$

$\sqrt{37}$ ✓

$\sqrt{37}>\sqrt{29}$ $\therefore$ outside.

1



Let C be the midpoint of the line PQ and the centre of the circle.

$C: \left(\frac{4+9}{2}\right), \left(\frac{1+6}{2}\right)$

$C: \left(\frac{13}{2}, \frac{7}{2}\right)$

The circle has equation

$\left(x-\frac{13}{2}\right)^2+\left(y-\frac{7}{2}\right)^2=r^2$

The length of the radius can be found from CR as R lies on the circle:

$r=\sqrt{\left(6-\frac{13}{2}\right)^2+\left(7-\frac{7}{2}\right)^2}$

$r=\frac{5\sqrt{2}}{2}$ or $r^2=\frac{25}{2}$

For PQ to be a diameter $PQ=5\sqrt{2}$ as it's twice the radius.

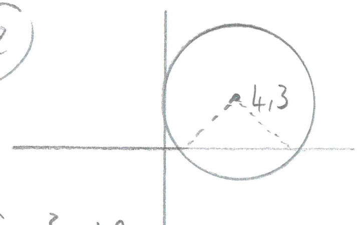
$PQ=\sqrt{(9-4)^2+(6-1)^2}$

$=\sqrt{5^2+5^2}$

$=\sqrt{50}$

$=5\sqrt{2}$ ✓

2



a $r^2=16$

b $(x-4)^2+(0-3)^2=16$

$(x-4)^2+9=16$

$(x-4)^2=7$

$x=4\pm\sqrt{7}$

$\therefore (4+\sqrt{7})-(4-\sqrt{7})$

$=0+2\sqrt{7}$ ✓

c $\frac{3\times2\sqrt{7}}{2}=3\sqrt{7}$

Pure 35 Algebraic Fractions

① $\frac{4x(3x^3+x^2+2)}{2x}$

$= 6x^3+2x^2+4$

② $\frac{(x+2)\cancel{(x+2)}}{\cancel{(x+2)}} \equiv x+2$

$a=1,\ b=2$

③ $\frac{\cancel{(x-4)}(x+3)}{\cancel{(x-4)}} \equiv x+3$

① $\frac{\cancel{(x+2)}(6x+1)}{2\cancel{(x+2)}} = \frac{6x+1}{2}$

② $\frac{6\cancel{(x^2-1)}}{x\cancel{(x^2-1)}} \equiv \frac{6}{x}$

③ $\frac{\cancel{(2x-5)}(x+3)}{\cancel{(2x-5)}(x-4)} \equiv \frac{x+3}{x-4}$

Yes, it factors.

⑥ No! You cannot cancel $x+A$ to A. It must be the whole factor rather than part if $\pm$ is between the terms

① $\frac{\cancel{(Ax^2+By^2)}(Ax^2-By^2)}{\cancel{(Ax^2+By^2)}}$

$\equiv Ax^2-By^2$

② $\frac{x^2(144-25x^2)}{2x(5-12x)}$

$\frac{x^2\cancel{(12-5x)}(12+5x)}{-2\cancel{(12-5x)}}$

$-\frac{x^2}{2}(12+5x)$

③ $\frac{(A+1)^{29}(A+1-1)}{2(A+1)} \equiv \frac{(A+1)^{28}(A)}{2\cancel{(A+1)}}$

$\frac{(A+1)^{28}(A)}{2} \equiv \frac{A}{2}(A+1)^{28}$