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A LEVEL MATHS

YEAR 1 PURE

ANSWERS

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(1) Indices
(2) Expanding Brackets
(3) Factorising Expressions
(4) More Indices (Negative and Fractional)
(5) Working with Surds
(6) Solving Quadratic Equations
(7) Completing the Square for Quadratics Expressions
(8) Function Notation
(9) Sketching Quadratic Graphs
(10) The Discriminant for Quadratic Equations
(11) Applications of Quadratics Equations
(12) Solving Linear Simultaneous Equations
(13) Linear & Non-Linear Simultaneous Equations
(14) Graphing Simultaneous Equations
(15) Linear Inequalities
(16) Quadratic Inequalities
(17) Graphing Inequalities
(18) Shading Inequalities
(19) Cubic Graphs
(20) Quartic Graphs
(21) Reciprocal Graphs
(22) The Intersection of Graphs
(23) Transforming Graphs (Translations)
(24) Transforming Graphs (Stretching/Reflecting)
(25) Straight Line Graphs in the form $y = mx + c$
(26) More Straight Line Graphs
(27) Straight Line Graphs (Parallel & Perpendicular)
(28) The Geometry of Straight Lines
(29) The Application of Linear Graphs
(30) Circle Geometry Midpoint & Perpendicular
(31) The Equation of a Circle
(32) Circles and Straight Lines (Intersections)
(33) Circles (Tangents and Chords)
(34) Circles and Triangles
(35) Algebraic Fractions
(36) Polynomial Division
(37) The Factor and Remainder Theorem
(38) An Introduction to Mathematical Proof
(39) Methods of Proof
(40) Binomial Expansion (Using Pascal's Triangle)
(41) Binomial Expansion (Factorial Notation)
(42) Binomial Expansion (The $\binom{n}{r}$Method)
(43) Binomial Expansion (Problem Solving)
(44) Binomial Expansion (Estimations and Approximations)
(45) The Cosine Rule
(46) The Sine Rule
(47) Areas of a Triangles
(48) Triangles (Problem Solving)
(49) Sine, Cosine & Tangent Graphs
(50) Transforming Graphs (Trigonometry)
(51) The 'CAST' Diagram for Trig Ratios
(52) Trigonometry (Exact Values)
(53) Proving Trigonometric Identities
(54) Solving Basic Trigonometric Equations
(55) More Challenging Trigonometric Equations
(56) Using Identities to Solve Trig Equations
(57) Vectors (Introduction)
(58) Vector Notation (Column and i and j form)
(59) Vectors (Magnitude and Direction)
(60) Vectors (Position and Direction Vectors)
(61) Vector Geometry
(62) Application of Vectors
(63) Differentiation (Gradients of Curves)
(64) Differentiation from 1st Principles
(65) Differentiating x^n (Basic Powers of)
(66) Differentiation (Quadratic Expression)
(67) Differentiation (Multiple Terms)
(68) Differentiation (Gradients, Tangents and Normals)
(69) Differentiation (Increasing and Decreasing Functions)
(70) Differentiation (Stationary Points)
(71) Differentiation (Gradient Functions
(72) The Applications of Differentiation
(73) Integration (Basic Expressions (x^n))
(74) Indefinite Integrals
(75) Integration (Finding c and Finding Functions)
(76) Integration (Definite Integrals)
(77) Integration (Basic Areas Under Curves)
(78) Integration ('Negative and Positive Areas')
(79) Integration (Areas between Curves and Lines)
(80) Basic Exponential Functions
(81) 'The' Exponential Function $y = e^x$
(82) Applications of Basic Exponential Models
(83) Logarithms (Simplifying & Evaluating)
(84) Logarithms (The Log Laws)
(85) Logarithms (Log and Exponential Equations)

Pure (26) Equations of Straight Lines

① $y=2x+1$ ①

$4x-y-8=0$ ②

$y=4x-8$ ②

① = ② Simultaneous Equations

$2x+1=4x-8$

$9=2x$

$x=\frac{9}{2}$

$\therefore y=10$

$\left(\frac{9}{2}, 10\right)$

② $5x+10y=20$

$10y=-5x+20$

$y=-\frac{1}{2}x+2$

③ when $x=0$

$-8y+8=0$

$8y=8$ $\therefore (0,1)$

$y=1$

When $y=0$

$6x+8=0$

$6x=-8$

$x=-\frac{8}{6}$

$x=-\frac{4}{3}$

$\therefore \left(-\frac{4}{3}, 0\right)$

① Sub in values

$q=-4x-8$ ①

$q=5(-4)-12$ ② Solve for q

$\therefore q=-32$

Sub into ①

$-32=-4x-8$

$4x=24$

$x=6$

② $y=-\frac{1}{2}x+3$

$2y=-x+6$

$x+2y-6=0$

③ $y=2(x-3)+1$

$0=2(x-3)+1$

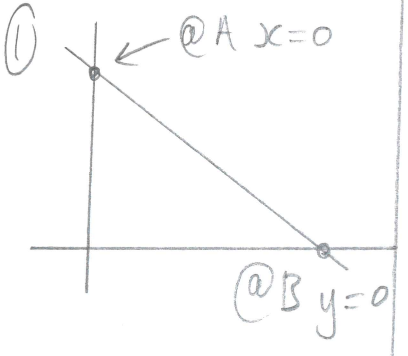
$-1=2(x-3)$

$-\frac{1}{2}=x-3$

$x=\frac{5}{2}$

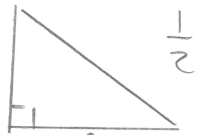
$\therefore (5/2, 0)$

①



A $x=0$	B $y=0$
$by=c$	$ax=c$
$y=\frac{c}{b}$	$x=\frac{c}{a}$
$\left(0,\frac{c}{b}\right)$	$\left(\frac{c}{a},0\right)$

$\frac{c}{b}$



$\frac{c}{a}$

$\frac{1}{2}bh$

$\frac{1}{2}\times\left(\frac{c}{b}\right)\left(\frac{c}{a}\right)=\frac{c^2}{2ab}$.

② x must be 0 for both lines on the y axis.

$\therefore px+qy=r$

$0+qy=r$

$y=\frac{r}{q}$

$\left(0, \frac{r}{q}\right)$

and

$y=mx+c$

$y=c$

$(0, c)$

$c=\frac{r}{q}$

$r=cq$

③ $p=3$, $q=2$

Pure 27 Parallel and Perpendicular Lines

1 (a) 3 (b) $-\frac{1}{3}$

2 If parallel

$m_1 = m_2$

If perpendicular

$m_1 = -\frac{1}{m_2}$

$y - x = 4$

$y = x + 4 \quad \therefore m_1 = 1$

$5x + 2y = 8$

$2y = -5x + 8$

$y = -\frac{5}{2}x + 4 \quad \therefore m_2 = -\frac{5}{2}$

parallel gradient $= 1$

perpendicular $= -1$

$\therefore$ neither.

3 If perpendicular

$m = -\frac{5}{2}$

$\therefore y - 3 = -\frac{5}{2}(x - 3)$

$y = -\frac{5}{2}x + \frac{15}{2} + 3$

$y = -\frac{5}{2}x + \frac{21}{2}$

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1 $y = 4x + 7$

$m_1 = 4$

If perpendicular

$m_2 = -\frac{1}{4}$

$ax - 2y + 8 = 0$

$2y = ax + 8$

$y = \frac{a}{2}x + 8$

$m_2 = \frac{a}{2}$

$\therefore \frac{a}{2} = -\frac{1}{4}$

$a = -\frac{1}{2}$

2 Gradient

$\frac{1 - 5}{5 - -1} = -\frac{4}{6} = -\frac{2}{3}$

$\therefore \perp m = \frac{3}{2}$

Midpoint

$\left(\frac{-1+5}{2}, \frac{5+1}{2}\right) = (2, 3)$

$y - 3 = \frac{3}{2}(x - 2)$

$y - 3 = \frac{3}{2}x - 3$

$y = \frac{3}{2}x$

3 $m = \frac{6}{-a}$

$\therefore p = \frac{6}{-a}$ o.e

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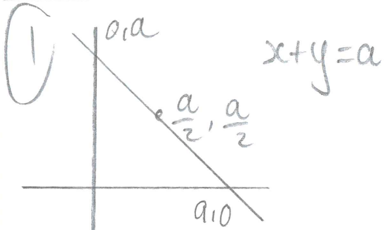
1 

$m = -1$, midpoint is $\left(\frac{a}{2}, \frac{a}{2}\right)$.

$py = qx + r$ has gradient 1

$py = qx + r$

$y = \frac{q}{p}x + \frac{r}{p}$

$\therefore \frac{q}{p} = 1$

$q = p$ ✓

The point $\frac{a}{2}, \frac{a}{2}$ lies on

$y = x + \frac{r}{p}$

$\therefore \frac{a}{2} = \frac{a}{2} + \frac{r}{p}$

$0 = \frac{r}{p}$

$\therefore r = 0$

2 If they don't intersect and they are not the same line then they are parallel.

$y = mx + c$ (1)

$x + py + q = 0$ (2)

$py = -x - q$

$y = -\frac{x}{p} - \frac{q}{p}$

$\therefore m \neq -\frac{1}{p}$

$m \in \mathbb{R}, m \neq -\frac{1}{p}$.

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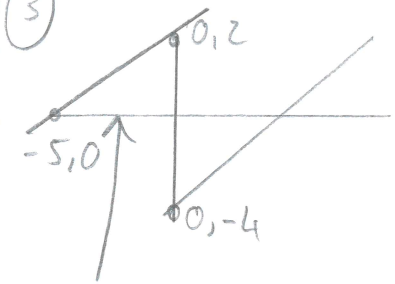
3 

$m = \frac{2 - 0}{0 - -5}$

$= \frac{2}{5}$

$y - -4 = \frac{2}{5}(x - 0)$

$y = \frac{2}{5}x - 4$

Loads of possible ways with this.

Steve Blades

28 Areas and Lengths

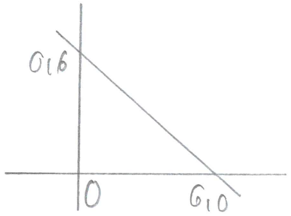
1. $d=\sqrt{(7--1)^2+(2-8)^2}$

$d=\sqrt{8^2+(-6)^2}$

$d=\sqrt{64+36}$

$d=10$

2.



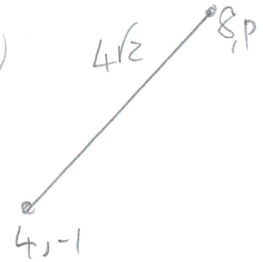
$A=\frac{1}{2}\times 6\times 6$

$=18$

$A=\frac{1}{2}\times 8\times 4$

$=16$

2.



$(8-4)^2+(p--1)^2=(4\sqrt{2})^2$

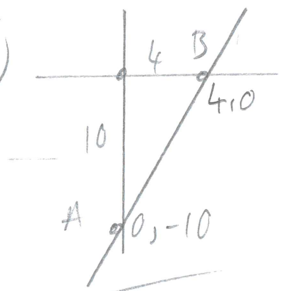
$16+(p+1)^2=32$

$(p+1)^2=16$

$p+1=\pm 4$

$\therefore p=-5$ or $p=3$

3.



$\sqrt{10^2+4^2}=\sqrt{116}$

$=2\sqrt{29}$

1. Sim Eq:

$y=x$ (1)

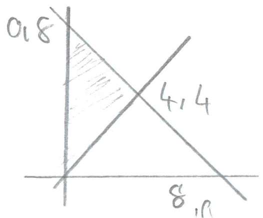
$x+y=8$ (2)

$x+x=8$ (2)

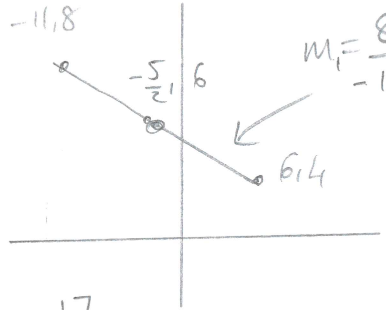
$x=4$

$\therefore y=4$

point $(4,4)$



1.



$m_1=\frac{8-4}{-11-6}$

$=\frac{-4}{17}$

$m_2=\frac{17}{4}$

Line equation

$y-6=\frac{17}{4}\left(x--\frac{5}{2}\right)$

$y=\frac{17}{4}x+\frac{133}{8}$

crosses y axis when $x=0$

$\therefore \frac{133}{8}$ (B)

crosses x axis when $y=0$

$\therefore -\frac{133}{34}$ (A)

Area $=\frac{1}{2}\times\frac{133}{34}\times\frac{133}{8}$

$=\frac{17689}{544}$

2. $\frac{q-4}{p-3}=1$ Gradient.

$q=p+1$

Length $(p-3)^2+(q-4)^2=36$

Simultaneous equations

$(p-3)^2+(p-3)^2=36$

$2(p-3)^2=36$

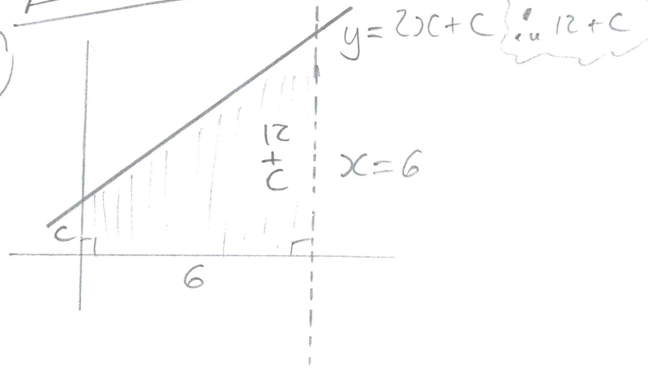
$(p-3)^2=18$

$p-3=\pm 3\sqrt{2}$

$p=3\pm 3\sqrt{2}$

$q=4\pm 3\sqrt{2}$

3.



$\frac{(c+12+c)}{2}\times 6=48$

$2c+12=16$

$2c=4$

$c=2$

29 Application of Straight Lines

1 (a) Cost of the painting in £ when it was first sold. or it's original value was £100.

(b) Pick a point (200,200) and (0,100)

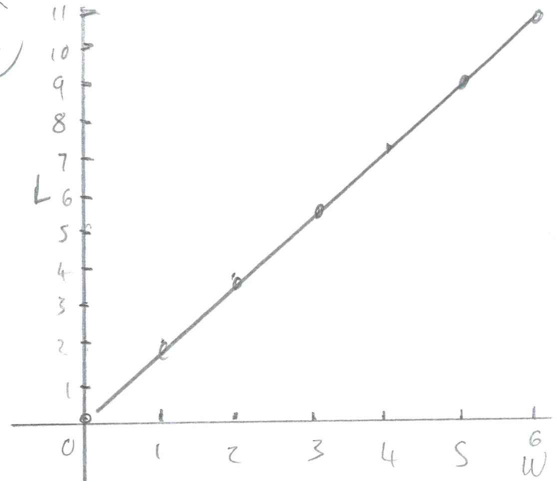
$m = \frac{200-100}{200}$

$= \frac{1}{2}$

(c) The painting increases by 50p each year after its first sold

(d) $V = \frac{1}{2}N + 100$

2 (a)



(b) Yes

(c) Starts at (0,0)

(d) $L = 1.8W$

(e) How many cm the leaf grows each week is a. b is 0 as it has no initial width at the start of the trial.

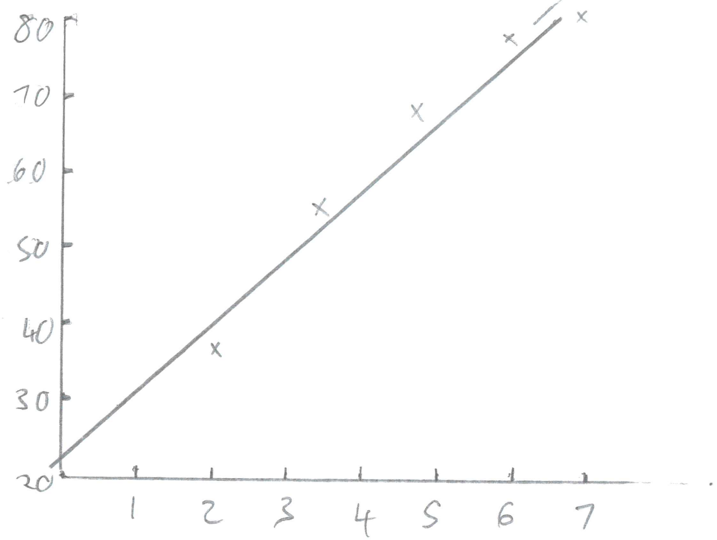
(f) The leafs width will be limited by its species and can't get infinitely large as time → ∞.

(g) $37 = 1.8w$

$20.555 = w$

∴ approx 21 weeks.

1



(a)

(b) Lots of answer! ← Use your graph.
Such as $P = 8M + 22$

(c) a is the increase in maths test % for each month of tuition

(d) The % increase in maths exam score without any tuition.

(e) (i) A student can't exceed 100%
(ii) A student won't study for an infinitly long period.

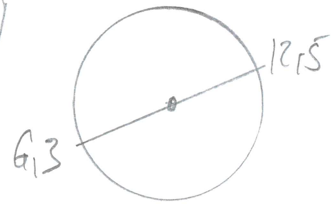
(f) doesn't start at (0,0)

(g) The students started with a lower base % in their maths test but progressed at a better rate as 12% per month increase v's ≈8% as found in (b).

Year 1 Circle Geometry
Midpoints and Perpendiculars

30

(1)



(a) $\frac{6+12}{2}, \frac{3+5}{2} = (9,4)$

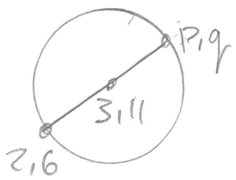
(b) $\sqrt{(12-6)^2+(5-3)^2}$

$\sqrt{6^2+2^2}$

$= 2\sqrt{10}$

(c) $\sqrt{10}$

(2)



$\frac{2+p}{2}=3$ and $\frac{6+q}{2}=11$

$\therefore 2+p=6$ $\quad 6+q=22$

$p=4$ $\quad q=16$

(3) $M_{diameter} = \frac{6--2}{6-0} = \frac{4}{3}$

$\therefore M$ of $\perp = -\frac{3}{4}$

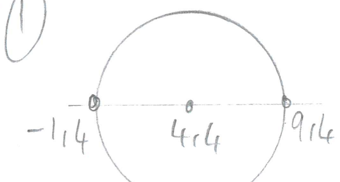
Straight line

$y-2=-\frac{3}{4}(x-3)$

$4y-8=-3x+9$

$3x+4y=17$

(1)

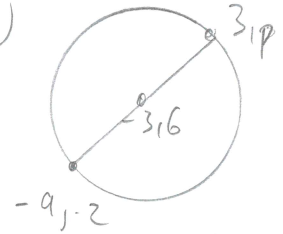


(a) $\frac{-1+9}{2}, \frac{4+4}{2} = (4,4)$

(b) $x=4$ as the diameter is part of the line $y=4$

$x=4$ and $y=4$ are $\perp$ or m of diameter $=0$ $\therefore$ the $\perp$ will have an infinite gradient.

(2)



$\therefore 100=(3+3)^2+(p-2)^2$

$100 = 36+p^2-4p+4$

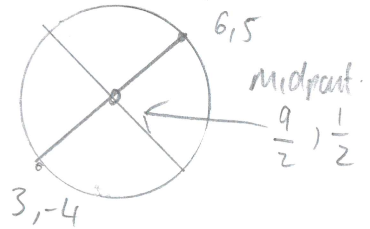
$p^2-4p-60$

$(p-10)(p+6)=0$

$p=10$ $\quad p\neq-6$

as centre is $(-3,6)$

(3)



$M_{PQ}=\frac{9}{3}$ $\therefore M=-\frac{1}{3}$

The perpendicular diameter gradient

Straight line through centre

$y-\frac{1}{2}=-\frac{1}{3}(x-\frac{9}{2})$

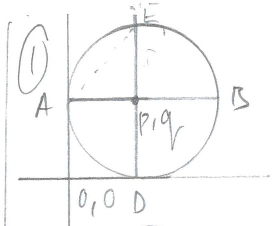
$y=-\frac{1}{3}x+\frac{3}{2}+\frac{1}{2}$

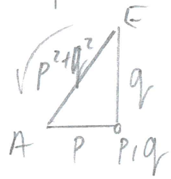
$y=-\frac{1}{3}x+2$

$3y=-x+6$

$3y+x-6=0$

(1)





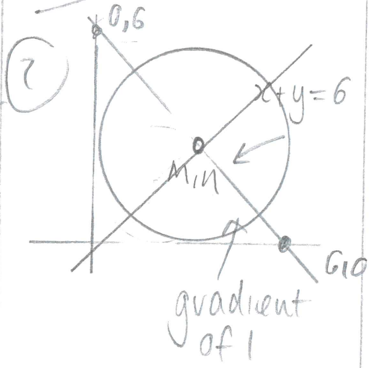
$AE=EB=BD=DA$

$\therefore$ a square

$\sqrt{p^2+q^2} \times \sqrt{p^2+q^2}$

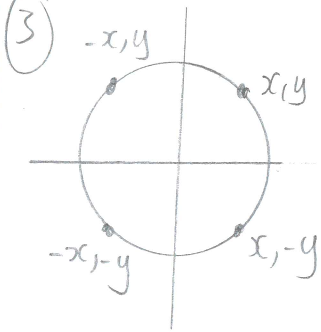
p^2+q^2

(2)



The centre of the circle will be the midpoint. The midpoint of $(6,0)$ and $(0,6)$ is $(3,3)$ $\therefore m=n$

(3)



(a) any of the above!

(b) Diameter

$\sqrt{(2x)^2+(2y)^2}$

$\sqrt{4x^2+4y^2}$

$2\sqrt{x^2+y^2}$

or just double $\sqrt{x^2+y^2}$

$2\sqrt{x^2+y^2}$

This is easier.