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A LEVEL MATHS

YEAR 1 PURE

ANSWERS

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(11) Applications of Quadratics

(1) Initial velocity ∴ $t=0$

(a) $V=3$

(b) Sub in $t=2$

$V=-(4)+8(2)+3$

$V=15$

(c) NOT using differentiation

(d) Solve $V=0$ ← Stationary ∴ velocity $=0$

$0=-t^2+8t+3$

$0=t^2-8t-3$

Using equation:

$t=8.358\ldots$ $t\neq-0.35$

↑ Stationary at 8.4 seconds but the model is only valid for $0\leq t\leq 3$

* You can use differentiation here!

(1) $t=0$ for initial

(a) height ∴ sub in $t=0$

$h=0+0$

$h=0m$

(b) $h=18$ C/B

$18=-t^2+10t$

$t^2-10t+18=0$

$t=7.65$ secs $t=2.35$ seconds

∴ 2.35 seconds

(c) You can differentiate here if you have learned it, otherwise complete the square

$h=-[t^2-10t]$

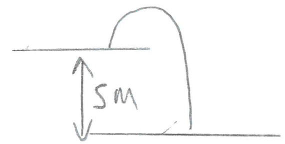
$=-[(t-5)^2-25]$

$=-(t-5)^2+25$

∴ 25m

(d) $t>10$ as it would give a negative height

(1)



(a) $t=0$, $K=5$

(b) $h=0$ ∴

$-2t^2+2t+5=0$

$2t^2-2t-5=0$

$t=2.16$ $t\neq-1.16$

∴ 2.16 seconds

(c) Max height. You can use differentiation or complete the square

$h=-2t^2+2t+5$

$=-2[t^2-t]+5$

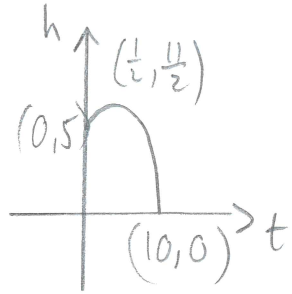
$=-2[(t-\frac{1}{2})^2-\frac{1}{4}]+5$

$=-2(t-\frac{1}{2})^2+\frac{1}{2}+5$

$t=\frac{1}{2}$ and height $=5.5m$.

(d) The impact with the water will decrease the velocity of the diver.

(c)



* finding max and min values is often quicker using differentiation which you learn later.

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Pure (12) Linear Simultaneous Equations

(1) $4x+10y=28$ ×2

$15x-10y=10$ ×5

(1)+(2) $19x=38$

$x=2$

$4(2)+10y=28$

$10y=20$

$y=2$

E/D

(2) Set $y=y$ ∴

$7x-11=2x+4$

$5x=15$

$x=3$

Sub in $y=2(3)+4$

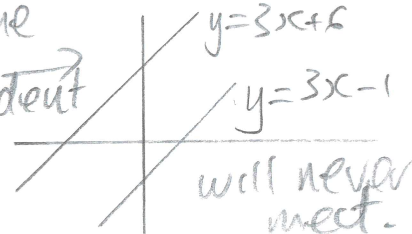
$y=10$

(3) Set $y=y$ ∴

~~3x~~ $+6$ = ~~3x~~ -1

$6\neq-1$ ∴ no solutions

Same gradient



will never meet.

(1) $0.1x+4y=9$

$1.2x-4y=4$

$1.3x=13$

$x=10$

Sub in:

$0.1(10)+4y=9$

$4y=8$

$y=2$

(2) (×2) $6y=12-6x$

(×3) $6y=15x-9$

∴ $15x-9=12-6x$

$21x=21$

$x=1$

$2y=5(1)-3$

$y=1$

(3) The gradients of the lines must be the same when in the form $y=mx+c$

C/B

(1) $py=-3x+14$

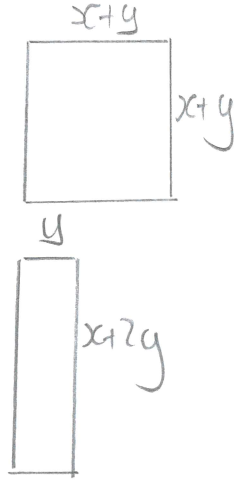
$y=-\frac{3}{p}x+\frac{14}{p}$

(2) $7y=2x+9$

$y=\frac{2}{7}x+\frac{9}{7}$

∴ $\frac{-3}{p}=\frac{2}{7}$

$p=\frac{-21}{2}$



∴ $4(x+y)=24$

(1) $x+y=6$

∴ $2y+2(x+2y)=16$

$2x+6y=16$

(2) $x+3y=8$

(2)−(1) $2y=2$

$y=1$

∴ $x=5$

Area of square

$=36$

Area of rectangle

$=7$

∴ (29)

(2) Sub in

$\frac{1}{2}pq+7p=26$ (1)

$2p-7+q=0$ (2)

∴ (2) $q=7-2p$

(1) $\frac{1}{2}(p)(7-2p)+7p=26$

$7p-2p^2+14p=52$

$2p^2-21p+52=0$

$p=\frac{13}{2}$ or $p=4$ ✓

∴ $q=-1$

(3) Anything where the gradient of the 2 lines are equal!

eg $6y=x-8$

$6y=x+12$

or

$y=9x+7$

$y=4+9x$

(13) Linear and Non-Linear Simultaneous Equations

(1) $y=y$ ∴

$x^2+5x-13=2x-3$

$x^2+3x-10=0$

$(x+5)(x-2)=0$

$x=-5$ or $x=2$

↓ ↓

$y=-13$ $y=1$

(2) Sub (1) into (2) D/E

$x(x-1)=6$

$x^2-x=6$

$x^2-x-6=0$

$(x-3)(x+2)=0$

↓ ↓

$x=3$ $x=-2$

∴ $y=2$ $y=-3$

(3) Sub (1) into (2)

$x^2+(x^2)=72$

$2x^2=72$

$x^2=36$

$x=\pm 6$

∴ $y=\pm 6$

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(1) (1) $x=10-2y$

(2) $(10-2y)y=8$

$10y-2y^2=8$

$2y^2-10y+8=0$

$y^2-5y+4=0$

$(y-1)(y-4)=0$

$y=1$, $y=4$

∴ $x=8$ $x=2$

(2) $y=\frac{7-5x}{3}$

$x^2+\left(\frac{7-5x}{3}\right)^2=5$

$x^2+\frac{49-70x+85x^2}{9}=5$

$9x^2+49-70x+25x^2=45$

$34x^2-70x+4=0$

$x=2$ or $x=\frac{1}{17}$

$y=-1$ or $y=\frac{38}{17}$

(3) Sub in: B/C

$(x-8)^2+(10)^2=4$

$(x-8)^2+100=4$

$(x-8)^2\neq -96$

∴ no real roots!

N.B. You can use the discriminant too!

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(1) $x^2+y^2=50$ (1)

$y=-x$ (2)

∴ (2) into (1) A/A*

$x^2+(-x)^2=50$

$2x^2=50$

$x^2=25$

$x=\pm 5$ c b $(5,-5)$

∴ $y=\mp 5$ $(-5,5)$ a d

(2) (1) $x=\frac{13-3y}{2}$

(1) $=\left(\frac{13-3y}{2}\right)y+2y^2=-30$

$13y-3y^2+4y^2=-30$

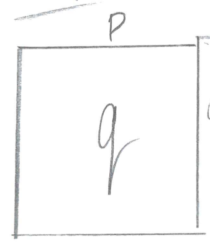
$y^2+13y+30=0$

$(y+3)(y+10)=0$

↓ ↓

$y=-3$ $y=-10$

∴ $x=1$ $x=\frac{43}{2}$

(3) 

∴ $p^2=q$ (1)

and $4p=\frac{2q}{3}$ (2)

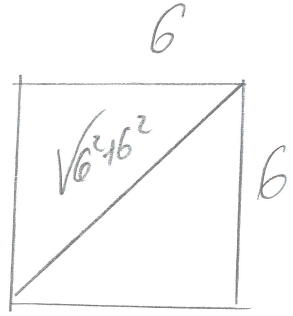
$q=6p$ (2)

∴ $p^2=6p$ (1)

$p^2-6p=0$

$p(p-6)=0$

$p\neq 0$ ∴ $p=6$



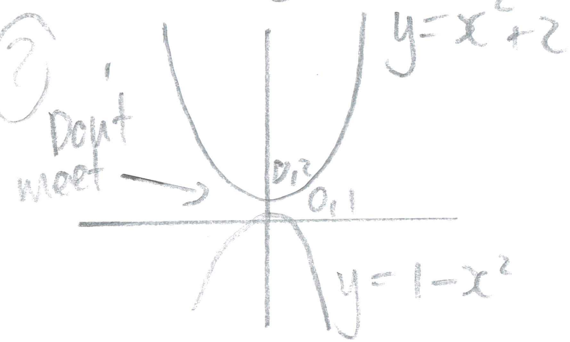
$\sqrt{6^2+6^2}=6\sqrt{2}$

(14) Simultaneous Equations on Graphs

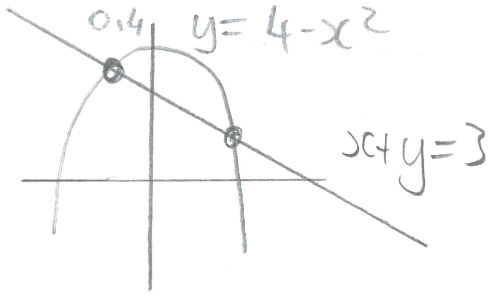
(1) Point of intersection are the solutions. $(3,-1)$

$\therefore x=3,\ y=-1$

(2)



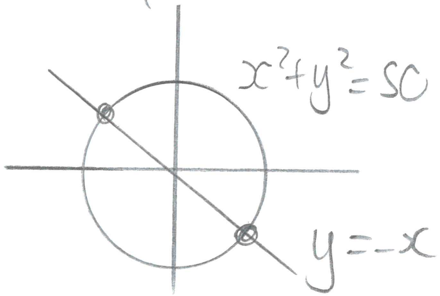
(1)



2 solutions as two points of intersection

(2)

(a)



(b) $x^2+(-x)^2=50$ (1)

$2x^2=50$

$x^2=25$

$x=\pm 5$

$\therefore y=\mp 5$

Points are $(5,-5)$ and $(-5,5)$

(3) $x^2-y=3$ (1)

$y=5-x$ (2)

Sub in (2) into (1)

$x^2-(5-x)=3$

$x^2+x-5-3=0$

$x^2+x-8=0$

$a=1,\ b=1,\ c=-8$

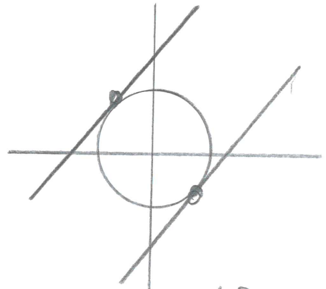
b^2-4ac

$1-4(1)(-8)=33$

$\therefore$ 2 real roots as $b^2-4ac>0$

Two points of intersection.

(1)



Touches $\therefore b^2-4ac=0$ as repeated root

(1) $x^2+y^2=30$

(2) $y=2x+k$

(1) $x^2+(2x+k)^2=30$

$x^2+4x^2+4kx+k^2=30$

$5x^2+4kx+k^2-30=0$

↑a ↑b ↑c

$b^2-4ac=0$

$(4k)^2-4(5)(k^2-30)=0$

$16k^2-20k^2+600=0$

$-4k^2+600=0$

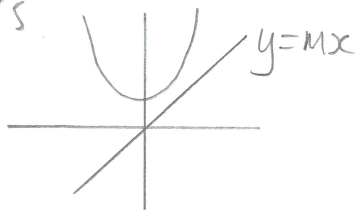
$k^2=150$

$k=\pm\sqrt{150}$

$k=\pm 5\sqrt{6}$

(2) Do not intersect

$\therefore b^2-4ac<0$ as no real roots



$3x^2+k=mx$

$3x^2-mx+k=0$

$b^2-4ac<0$

$m^2-4(3)k<0$

$(m+2\sqrt{3k})(m-2\sqrt{3k})<0$

$\therefore -2\sqrt{3k}<m<2\sqrt{3k}$

(3) $-2\left[t^2-\frac{k}{2}t\right]$

$-2\left[\left(t-\frac{k}{4}\right)^2-\frac{k^2}{16}\right]$

$-2\left(t-\frac{k}{4}\right)^2+\frac{k^2}{8}$

$\therefore \frac{k^2}{8}=30$

$k^2=240$

$k=\pm 4\sqrt{15}$

$\therefore +4\sqrt{15}$

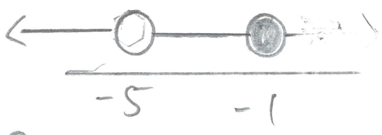
Pure (15) Linear Inequalities

① $4x < 12$

$x < 3$

② $2x < 10$

(a) $x < 5$



(b) $-4, -3, -2, -1$

③ $-0.1x \le 5$

$x \ge \frac{5}{-0.1}$

$x \ge -50$

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① (×2) $6 - 4x > -6$ (×20)

$12 > 4x$

$3 > x$

② $\cancel{x^2} - x < \cancel{x^2} - 8$

$-x < -8$

$x > 8$

③ $3 - 6x \le 0$

$3 \le 6x$

$\frac{1}{2} \le x$

$-8 < 2x + 14 < 24$

$-22 < 2x < 10$

$-11 < x < 5$

$\therefore$

$\frac{1}{2} \le x < 5$

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① $x \le \frac{1}{2k}$ ①

$12x - 24 \ge x$

$11x \ge 24$

$x \ge \frac{24}{11}$

$\frac{1}{2k} = \frac{24}{11}$

$k = \frac{11}{48}$ $\therefore k \ge \frac{11}{48}$

② $6 \le kx + 1 < 10$

$5 \le kx < 9$

$\frac{5}{k} \ge x > \frac{9}{k}$

$\therefore \frac{9}{k} < x \le \frac{5}{k}$

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